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Fractional-Step Block Method For Direct Solution Of Third Order Ordinary Differential Equations (IVPS)

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Abstract

This article produced a one-eight linear multi-step method for the numerical integration of third-order initial value problems (IVPs) of ordinary differential equations (ODEs). The method was achieved by considering the power series polynomial as an approximate solution using the techniques of interpolation and collocation. The resulting equations were solved for the unknown parameters and substituted into the approximate solution to the problem to obtain the required discrete and additional formulas that constituted the proposed block method. Analysis of the basic properties of the method reveals that it has theoretical order five, zero stable, consistent, convergence, and absolute stability. The numerical experiment results showed that the method compares well with the three cited methods in literature and has the potential to solve non-linear third-order ODEs.

Keywords: *power series, grid points, off-grid points, convergence, interpolation, collocation AMS Subject classification: 65L05, 65L06,.*

1. Introduction

Ordinary Differential Equations (ODEs) are widely used in the fields of management, engineering, science, technology, and social sciences. These physical occurrences are easier to understand when they are expressed as mathematical equations. Many of these mathematical formulas result in ODEs of various degrees and orders. For instance, the authors in [1, 4] used fractional order system of ODEs to analyse the transmission dynamics of infectious diseases. In this paper, we consider higher-order ordinary differential equations of the form:

$$y^{(3)} = f\left(x, y, y', y''\right), \quad y(x_0) = \alpha_0, \quad y'\left(x_0\right) = \alpha_1, \quad y''(x_0) = \alpha_2 \qquad (1)$$

where is a given real value function which is continuous within the interval of integration.

An equation (1) is often solved numerically by reducing it to a system of first-order ODEs. Then the resulting system of equations is solved by appropriate existing methods of solving first-order ordinary differential equations. Furthermore, several authors have adopted the reduction method for solving general solutions of higher-order ODEs, most notably Ref. [5]–Ref. [7]. Also, Awoyemi [4] claimed that the reduction method for the first-order system is not cost-effective because of computer time and computational effort. Remarkably, authors such as Ref. [9]–Ref. [14], to name a few, have worked to provide methods for directly solving higher-order initial value problems (IVPs) instead of translating higher-order ODEs to first-order systems. The hybrid numerical method with block extension was used by Duromola and Momoh [15] to obtain the direct solution of a third-order IVP of ODEs. The method has an order of accuracy of five. This work presents an order-five, one-eight linear, multi-step method for the direct solution of general third-order ODEs.

2. Methodology/Derivation of the proposed method

Power series of the form:

$$y\left(x\right) = \sum_{j=0}^{(r+s)-1} a_j x^j \qquad (2)$$

is considered as basis function.

The third derivative of (3) gives:

$$y^{(3)}(x) = \sum_{j=0}^{(r+s)-1} j\left(j-1\right)\left(j-2\right) a_j x^{j-3} \qquad \textbf{(3)}$$

Equating (3) and (1) yields the differential system:

$$\sum_{j=0}^{(r+s)-1} j\left(j-1\right)\left(j-2\right) a_j x^{j-3} = f(x, y(x), y'(x), y''(x)) \qquad (4)$$

where a_j's the parameters to be determined, r and s denotes the number of collocation and interpolation points respectively. By collocating Eq. (4) at the mesh points $x = x_{n+j}$, $j = 0\left(\frac{1}{32}\right)\frac{1}{8}$, and interpolating Eq. (2) at $x = x_{n+j}$, $j = \frac{1}{32}, \frac{1}{16}, \frac{3}{32}$ yields a system of equations for collocation equation,

$$\sum_{j=0}^{(r+s)-1} a_j x^j = y_{n+j}, \quad j = \frac{1}{32}, \frac{1}{16}, \frac{3}{32} \qquad (5)$$

and for interpolation,

$$\sum_{j=0}^{(r+s)-1} j(j-1)(j-2)(j-3)a_j x^{j-4} = f_{n+j}, \quad j=0\left(\frac{1}{32}\right)\frac{1}{8} \tag{6}$$

By putting these equation systems into matrix form and solving them to determine the parameter values $a_j's$, $j=0\left(\frac{1}{32}\right)\frac{1}{8}$,

Thus, after a few simplifications, this provides a continuous hybrid linear scheme with continuous coefficients of this kind Eq. (8) when substituted in Eq. (2):

$$y(t)=\sum_{j}^{i}\alpha_j y_{n+j}(t)+h^3\sum_{j}^{i}\beta_j f_{n+j}(t) \tag{7}$$

The coefficient of $\alpha_j(x)$ and $\beta_j(x)$ are:

$\alpha_{\frac{1}{32}}(t) = 512t^2 - 80t + 3$ $\quad\alpha_{\frac{1}{16}}(t) = -1024t^2 + 128t - 3$ $\quad\alpha_{\frac{3}{32}}(t) = 512t^2 - 48t + 1$

$$\beta_0(t)=\frac{1}{165150720}\begin{pmatrix}34359738368t^7 - 18790481920t^6 + 4110417920t^5 - 458752000t^4\\ +27525120t^3 - 845824t^2 + 10832t - 21\end{pmatrix}$$

$$\beta_{\frac{1}{32}}(t)=\frac{-1}{41287680}\begin{pmatrix}34359738368t^7 - 16911433728t^6 + 3053453312t^5\\ -220200960t^4 + 854784t^2 - 41920t + 609\end{pmatrix}$$

$$\beta_{\frac{1}{16}}(t)=\frac{1}{27525120}\begin{pmatrix}34359738368t^7 - 15032385536t^6 + 22313698728t^5\\ -110100480t^4 - 114688t^2 + 19344t - 441\end{pmatrix}$$

$$\beta_{\frac{3}{32}}(t)=\frac{-1}{41287680}\begin{pmatrix}34359738368t^7 - 13153337344t^6 + 1644167168t^5\\ -73400320t^4 + 62720t^2 - 256t - 21\end{pmatrix}$$

$$\beta_{\frac{1}{8}}(t)=\frac{1}{165150720}\begin{pmatrix}34359738368t^7 - 11274289152t^6 + 1291845632t^5\\ -55050240t^4 + 43008t^2 + 80t - 21\end{pmatrix} \tag{8}$$

where $t=\frac{x-x_n}{h}$

Evaluate (7) at $t=0,\frac{1}{8}$ to obtain the discrete schemes:

$$y_n - 3y_{n+\frac{1}{32}} + 3y_{n+\frac{1}{16}} - y_{n+\frac{3}{32}} = \frac{-h^3}{7864320}\left(f_n + 116f_{n+\frac{1}{32}} + 126f_{n+\frac{1}{16}} - 4f_{n+\frac{3}{32}} - 4f_{n+\frac{1}{8}}\right) \tag{9a}$$

$$y_{n+\frac{1}{8}} - y_{n+\frac{1}{32}} + 3y_{n+\frac{1}{16}} - 3y_{n+\frac{3}{32}} = \frac{h^3}{7864320}\left(f_n - 4f_{n+\frac{1}{32}} + 126f_{n+\frac{1}{16}} + 116f_{n+\frac{3}{32}} + f_{n+\frac{1}{8}}\right) \tag{9b}$$

2.1 Implementation in block mode

The general block formula proposed by Awoyemi *et al*. [16], in the normalised form is given by

$$A^{(0)}Y_m = ey_n + h^{\mu-\lambda}df(y_m) + h^{\mu-\lambda}bF(y_m) \tag{10}$$

By evaluating Eq. (8) at $t=\frac{1}{8}$; the first and second derivatives at $x=x_{n+i}$, $i=0\left(\frac{1}{32}\right)\frac{1}{8}$ and substituting into Eq. (10) gives the coefficients matrices as

$$d=\begin{bmatrix}\frac{311}{110100480} & \frac{1303}{82575360} & \frac{853}{22020096} & \frac{659}{9175040} & \frac{179}{688128} & \frac{53}{92160} & \frac{147}{163840} & \frac{7}{5760} & \frac{251}{23040} & \frac{29}{2880} & \frac{27}{2560} & \frac{7}{720}\end{bmatrix}^T$$

$$e=\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{32} & \frac{1}{16} & \frac{3}{32} & \frac{1}{8} & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ \frac{1}{2048} & \frac{1}{512} & \frac{9}{2048} & \frac{1}{128} & \frac{1}{32} & \frac{1}{16} & \frac{3}{32} & \frac{1}{8} & 1 & 1 & 1 & 1 \end{bmatrix}^T$$

$A^{(0)} = 12\times 12$ identity matrix

$$b=\begin{bmatrix} \frac{-4337}{165150720} & \frac{11}{4128768} & \frac{793}{11010048} & \frac{3743}{20643840} & \frac{493}{2580480} & \frac{1}{640} & \frac{117}{40960} & \frac{1}{240} & \frac{323}{11520} & \frac{31}{720} & \frac{51}{1280} & \frac{2}{45} \\ \frac{-1867}{55050240} & \frac{-601}{13762560} & \frac{-831}{18350080} & \frac{-313}{13762560} & \frac{-1193}{1720320} & \frac{-1}{3072} & \frac{27}{81920} & \frac{1}{960} & \frac{-11}{960} & \frac{1}{120} & \frac{9}{320} & \frac{1}{60} \\ \frac{33}{3318350080} & \frac{25}{4128768} & \frac{731}{55050240} & \frac{263}{6881280} & \frac{17}{2580480} & \frac{73}{2580480} & \frac{3}{8192} & \frac{1}{720} & \frac{53}{11520} & \frac{1}{720} & \frac{47}{2880} & \frac{2}{45} \\ \frac{-131}{330301440} & \frac{-97}{82575260} & \frac{-271}{110100480} & \frac{-341}{82575360} & \frac{-199}{10321920} & \frac{-1}{30720} & \frac{-9}{163840} & 0 & \frac{-19}{23040} & \frac{-1}{2880} & \frac{-3}{2560} & \frac{7}{720} \end{bmatrix}^T$$

3. Analysis of the method

In this section, the analysis of the basic properties of the method was carried out as follows.

3.1 Order and Error Constant of the method

The formula in Eq. (9b) in a conventional linear multistep method can be express as

$$\sum_{j=1}^{3}\alpha_{\frac{j}{8}}y_{n+\frac{j}{32}} = h^3\sum_{j=0}^{4}\beta_{\frac{j}{8}}y'''_{n+\frac{j}{32}} \tag{11}$$

According to Lambert [5], the local truncation error associated with Eq. (11) was defined by the difference operator.

$$L_{\frac{j}{32}}\{y(x):h\} = \sum_{j=0}^{k}\left\{\alpha_{\frac{j}{32}}y\left(x_n+\frac{j}{32}h\right)-h^3\beta_{\frac{j}{32}}y'''\left(x_n+\frac{j}{32}h\right)\right. \tag{12}$$

$y(x)$ is assumed to have continuous derivative of a sufficiently high order. Therefore expanding (10b) in Taylor series about the point x to obtain the expression

$$L_{\frac{j}{32}}\{y(x):h\} = c_0y(x)+c_1hy'(x)+c_2h^2y''(x)+\ldots+c_{p+2}h^{p+2}y^{(p+2)}(x)+c_{p+3}h^{p+3}y^{(p+3)}(x) \tag{13}$$

The term c_{p+3} is called the error constant and implies that the local truncation error is given by:

$$t_{n+k} = c_{p+4}h^{(p+3)}y^{(p+3)}(x_n)+0h^{(p+4)} \tag{14}$$

Since $c_0 = c_1 = \ldots = c_{p+2} = 0, c_{p+3} \neq 0$. See Ref. [17]; the method has order $p = 5$ with error constant

$$c_{p+3} = \frac{19}{11083077207182080}$$

3.2 Zero Stability of the Method

According to Fatula [18], a block method is zero stable provided the roots $z_j,\ j=1(1)k$ of the first characteristic polynomial $\rho(r)$ specified as

$$\rho(z)=\det\left[\sum_{j=0}^{k}A^{(j)}Z^{k-j}\right]=0,\ A^{(0)}=-1 \qquad (15)$$

Satisfies $|z_j|\le 1,$ and for those roots with $|z_j|=1,$ the multiplicity must not exceed 2. By definition (3.2), the block is zero stable since the roots of the characteristic polynomial satisfy $|z|\le 1$ and the root $|z|=1$ has multiplicity not exceeding the order of the differential equation. Moreover, as $h^{\mu}\to 0, \rho(z)=z^{r-\mu}(\lambda-1)^{\mu}$, where μ is the order of the differential equation, for the block method, $r=12,$ and $\mu=3$

$$\rho(z)=\lambda^{9}(\lambda-1)^{3}=0$$

$$\Rightarrow \lambda=0,0,0,0,0,0,0,0,0,1,1,1$$

Hence, worth concluding that the method is Zero Stable

3.3 Consistency of the Method.

From Eq. (9b), the first and second characteristics polynomials of the method are given by

$$\rho(r)=r^{\frac{1}{8}}-3r^{\frac{3}{32}}+3r^{\frac{1}{16}}-r^{\frac{1}{32}}$$

$$\sigma(r)=\frac{1}{7864320}-\frac{4}{7864320}r^{\frac{1}{32}}+\frac{126}{7864320}r^{\frac{1}{16}}+\frac{116}{7864320}r^{\frac{3}{32}}+\frac{1}{7864320}r^{\frac{1}{8}}$$

This implies that the method presented in this report is consistent since it satisfies the following conditions:

i. The order of the method is $p=5>1$ which is obvious.

ii. For the method, $\alpha_1=1,\alpha_2=-3,\alpha_3=3\,and\ \alpha_4=-1$, thus

$\sum_{j=1}^{4}\alpha_j=1-3+3-1=0$, show the condition (ii) is satisfied.

iii. If $\rho(r)=r^{\frac{1}{8}}-3r^{\frac{3}{32}}+3r^{\frac{1}{16}}-r^{\frac{1}{32}}$ and $\rho'(r)=\frac{1}{8}r^{-\frac{7}{8}}-\frac{9}{32}r^{-\frac{29}{32}}+\frac{3}{16}r^{-\frac{15}{16}}-\frac{1}{32}r^{-\frac{31}{32}}$

It follows from here that $\rho(1)=0=\rho'(1)$

Show that the condition (iii) is satisfied as well

iv. Note that

$$\rho'''(r)=\frac{105}{512}r^{-\frac{23}{8}}-\frac{15921}{32768}r^{-\frac{93}{32}}+\frac{1395}{4096}r^{-\frac{47}{16}}-\frac{1953}{32768}r^{-\frac{95}{32}}$$

$$\Rightarrow \rho'''(1)=\frac{1}{512}=3!\sigma(1)$$

Thus, the condition (iv) is satisfied.

Hence the method is consistent

3.4 Convergence of the Method

According to Henrici [19], the necessary and sufficient condition for a numerical method to be convergent is to be Zero Stable and Consistent. Thus, since it has been successful shown in (3.2) and (3.3) above respectively. Hence, the method is said to convergent.

3.5 Region of Absolute Stability of the Method

Considering the stability polynomial in the general form:

$$\pi\left(r,\bar{\mathrm{h}}\right)=\rho\left(r\right)-\bar{\mathrm{h}}\,\sigma\left(r\right)=0 \qquad (16)$$

where, $\bar{\mathrm{h}}=h^2\lambda$ and $\lambda=\dfrac{\partial f}{\partial y}$ is assumed constant. The first and second characteristics polynomials of Eq. (9b) are given by

$$\rho\left(r\right)=r^{\frac{1}{8}}-3r^{\frac{3}{32}}+3r^{\frac{1}{16}}-r^{\frac{1}{32}}$$

$$\sigma\left(r\right)=\frac{1}{7864320}-\frac{4}{7864320}r^{\frac{1}{32}}+\frac{126}{7864320}r^{\frac{1}{16}}+\frac{116}{7864320}r^{\frac{3}{32}}+\frac{1}{7864320}r^{\frac{1}{8}}$$

The boundary of the region of the absolute stability is

$$\bar{\mathrm{h}}=\frac{\rho\left(r\right)}{\sigma\left(r\right)}=\frac{7864320\left(r^{\frac{1}{8}}-3r^{\frac{3}{32}}+3r^{\frac{1}{16}}-r^{\frac{1}{32}}\right)}{1-4r^{\frac{1}{32}}+126r^{\frac{1}{16}}+116r^{\frac{3}{32}}+r^{\frac{1}{8}}}=0 \qquad (17)$$

By setting $r=e^{i\theta}$, then Eq. (17) becomes

$$\bar{h}\left(\theta\right)=\frac{7864320\left(e^{\frac{i\theta}{8}}-3e^{\frac{3i\theta}{32}}+3e^{\frac{i\theta}{16}}-e^{\frac{i\theta}{32}}\right)}{1-4e^{\frac{i\theta}{32}}+126e^{\frac{i\theta}{16}}+116e^{\frac{3i\theta}{32}}+e^{\frac{i\theta}{8}}} \qquad (18)$$

Evaluate Eq. (18), and equate the imaginary part to zero gives

$$\bar{h}\left(\theta\right)=\frac{7864320\left(\cos\frac{1}{8}\theta-8\cos\frac{3}{32}\theta+28\cos\frac{1}{16}\theta-56\cos\frac{1}{32}\theta+35\right)}{2\cos\frac{1}{8}\theta+224\cos\frac{3}{32}\theta-424\cos\frac{1}{16}\theta+28448\cos\frac{1}{32}\theta+29350} \qquad (19)$$

Evaluating Eq. (19) at the interval of 30^0 gives the following results of the boundaries for the region of absolute stability of the method as tabulated below;

Table 1 Boundaries for region of absolute stability

θ	0^0	30^0	60^0	90^0	120^0	150^0	180^0
$\bar{h}\left(\theta\right)$	0	3.51×10^{-13}	8.98×10^{-11}	2.30×10^{-9}	2.29×10^{-8}	1.37×10^{-7}	5.89×10^{-7}

From table (1) above, it could be deduced that the region of absolute stability of the method is given by $x\left(\theta\right)=\left(0,5.89\times10^{-7}\right)$ which satisfies the condition for p-stability.

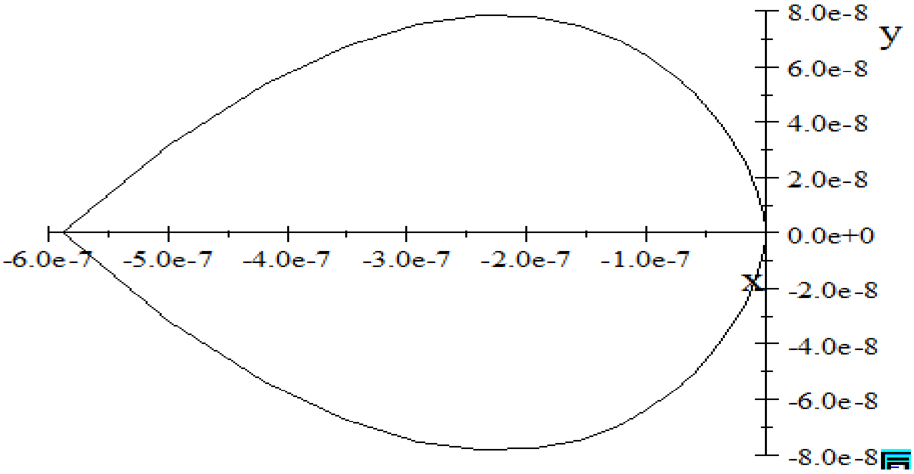

Fig. 1: Domain of stability (i.e. absolute) of the proposed method

4. Numerical Results

In this section, the method was utilized to solve some specific initial value problems of third-order ordinary differential equations to verify its accuracy and workability.

Problem 1.

$$y''' = x - 4y$$

$$y(0) = 0,\ y'(0) = 0,\ y''(0) = 1,\ \ h = 0.1$$

Exact solution: $y(x) = \dfrac{3}{16}(1 - \cos 2x) + \dfrac{1}{18}x$

Source: Anake et.al. [20]

Table 2: Result of Problem 1, computed with block method, *h* = 0.1

x-value	y-exact-solution	y-computed solution	Error in new method P=5	Error in Anake et.al. [20] P=5
0.1	0.00000488280	0.00000488280	2.546E-12	2.0952E-09
0.2	0.00001953106	0.00001953105	3.234E-12	1.6375E-08
0.3	0.00004394434	0.00004394434	1.209E-12	1.1154E-07
0.4	0.00007812195	0.00007812194	1.695E-12	9.8800E-07
0.5	0.00001953106	0.00001953105	3.233E-12	3.0406E-06
0.6	0.00004394434	0.00004394434	1.209E-12	9.0126E-06
0.7	0.00007812195	0.00007812194	1.694E-12	1.6965E-05
0.8	0.00012206285	0.00012206286	4.04E-12	2.6772E-05
0.9	0.00004394434	0.00004394434	1.207E-12	3.8135E-05
1.0	0.00007812195	0.00007812194	1.693E-12	5.0596E-05

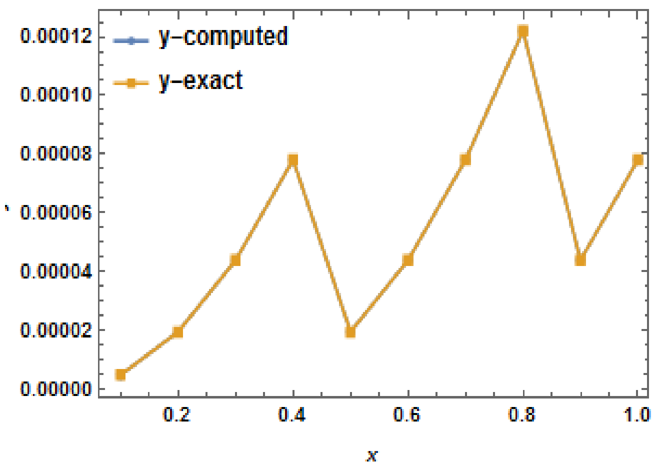

Figure 2: Comparison of absolute errors of the proposed method on problem 1 as compared with Exact solution

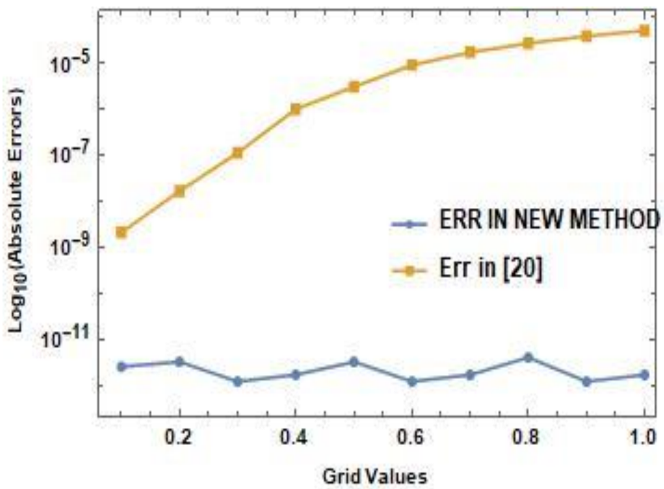

Figure 3: Comparison of absolute errors of the proposed method on problem 1 as compared with Anake *et. al* [20]

Problem 2.

$$y''' = 3\sin(x)$$

$$y(0) = 1,\ y'(0) = 0,\ y''(0) = -2,\ \ h = 0.1$$

Exact solution: $y(x) = 3\cos(x) + \frac{x^2}{2} - 2$

Source: Adoghe and Omole [21]

Problem 3.

$$y''' = e^x$$

$$y(0) = 3,\ y'(0) = -1,\ y''(0) = 5,\ \ h = 0.1$$

Exact solution: $y(x) = 2 + 2x^2 + e^x$

Source: Obarhua and Kayode [22]

Table 3: Result of Problem 2, computed with block method, *h* = 0.01

x-value	y-exact-solution	y-computed solution	Error in new method P=5	Error in Adoghe and Omole [21] P=5
0.1	0.99999843750	0.99999843750	0.0000	2.2204E-16
0.2	0.99999960937	0.99999960937	0.0000	4.4409E-16
0.3	0.99999990234	0.99999990234	1.0E-20	1.3323E-15
0.4	0.99999912109	0.99999912109	0.0000	3.8858E-15
0.5	0.99999755859	0.99999755859	0.0000	9.2149E-15
0.6	0.99999912109	0.99999912109	1.0E-20	1.8985E-14
0.7	0.99999960937	0.99999960937	2.0E-20	3.4084E-14
0.8	0.99999843750	0.99999843750	3.0E-20	5.7343E-14
0.9	0.99999648437	0.99999648437	1.0E-20	9.0095E-14
1.0	0.99999843750	0.99999843750	5.0E-20	1.3678E-13

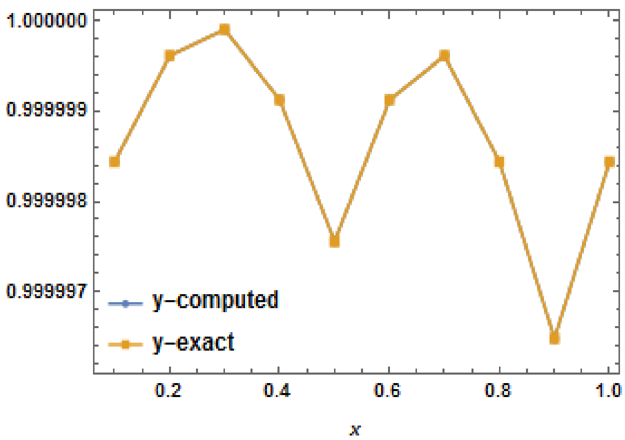

Figure 4 Comparison of absolute errors of the proposed method on problem 2 as compared with Exact solution

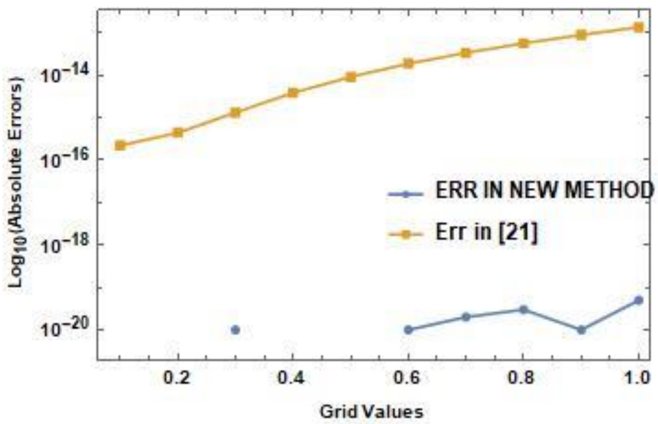

Figure 5: Comparison of absolute errors of the proposed method on problem 2 as compared with Adoghe and Omole [21]

Table 4: Result of Problem 3, computed with block method, *h* = 0.1

x-value	y-exact-solution	y-computed solution	Error in new method P=5	Error in Obarhua and Kayode [22] P=5
0.1	**3.0031494191527391486**	**3.0031494191527391486**	**1.5984E-18**	**4.65668E-11**
0.2	**3.0063476970037620101**	**3.0063476970037620101**	**4.653E-19**	**4.22858E-10**
0.3	**3.0095948642140710855**	**3.0095948642140710856**	**4.08239E-17**	**1.51196E-09**
0.4	**3.0128909515406343767**	**3.0128909515406343767**	**3.76471E-17**	**3.73730E-09**
0.5	**3.0063476970037620101**	**3.0063476970037620101**	**2.5861E-18**	**1.35178E-08**
0.6	**3.0095948642140710855**	**3.0095948642140710855**	**1.5729E-18**	**1.35178E-08**
0.7	**3.0128909515406343767**	**3.0128909515406343765**	**4.21813E-17**	**2.21617E-08**
0.8	**3.0162359898366857475**	**3.0162359898366857475**	**3.64115E-17**	**3.41303E-08**
0.9	**3.0095948642140710855**	**3.0095948642140710855**	**3.70003E-18**	**5.01217E-08**
1.0	**3.0128909515406343767**	**3.0128909515406343766**	**2.8074E-18**	**7.09074E-08**

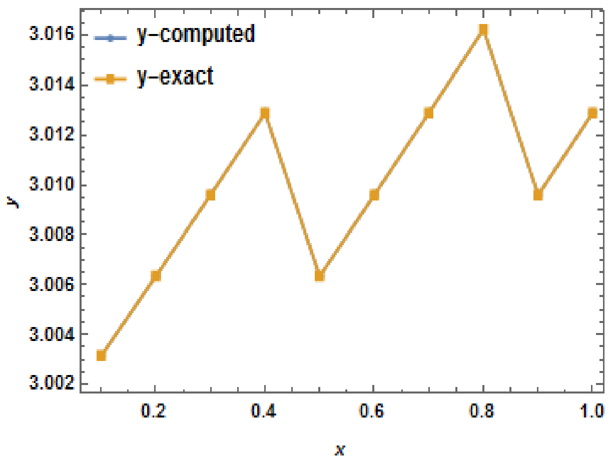

Figure 6: Comparison of absolute errors of the proposed method on problem 2 as compared with Exact solution

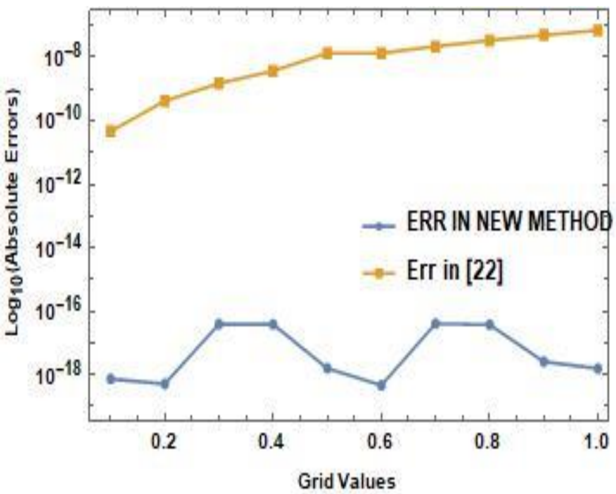

Figure 7: Comparison of absolute errors of the proposed method on problem 2 as compared with Obarhua and Kayode [20]

5.Discussion

In this research work, we have applied the procedures of collocation and interpolation to develop a linear hybrid multistep method for solving the initial value problem of third-order ordinary differential equations. Table 2, Table 3, and Table 4 displaced the results of the proposed method as applied to the test problems 1, 2 and 3. The results were further analysed using graphs displaced in Figures 2 to 7. It is clear from Tables 1 to 3 and Figures 2 to 7 that the derived method is better in terms of accuracy than the ones proposed by Anake et al. [20], Adoghe and Omole [21], and Obarhua and Kayode [22].

6. Conclusion

The fractional method has been developed to solve third-order ordinary differential equations directly. The main and additional formulas constituting the proposed block method were obtained from the same continuous scheme derived via interpolation and collocation procedures. The stability properties and region of the method were discussed. The method is applied in block form. Numerical results from the block method show they are efficient and adequate for solving general third-order initial value problems of ordinary differential equations. When the results were compared to those proposed by Anake et al. [20], Adoghe and Omole [21], and Obarhua and Kayode [22], the new results were better in terms of accuracy.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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