Abhath Journal of Basic and Applied Sciences., 2(2) (2023) 21-28

Abhath Journal of Basic and Applied Sciences

Website:https://ojs.abhath-ye.com/index.php/OJSRJBAS/about

doi: https://doi.org/10.59846/ajobas.v2i2.514

Research paper, Short communication, Review, Technical paper

Certain results for the three- variable Srivastava polynomials with two parameters

Salem Saleh Barahmah

Department of Mathematics, University of Aden, Yemen

***Corresponding author E-mail:** salemalqasemi@yahoo.com*

Received:25 Oct 2023. Accepted: 28 Nov 2023. Published 30 Dec 2023.

Abstract

The objective of this paper is to prove general theorems on generating functions involving two-parameter three-variable Srivastava polynomials, Laguerre polynomials, and two-variable Lagrange polynomials. Some applications of these theorems lead to a number of bilateral generating functions involving well-known classical polynomials.

MSC 2010 :33C45, 33C05, 33C65.

***Keywords**: Three-variable Srivastava polynomials with two parameters, Lagrange polynomials of two variables, Laguerre polynomials of two variables, generating functions..*

1. Introduction

The srivastava polynomials is defined by [10]:

$$S_n^N(x) = \sum_{m=0}^{\left[\frac{n}{N}\right]} \frac{(-n)_{Nm}}{m!} A_{n,m}\, x^m \quad (n \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}\,; N \in \mathbb{N}), \qquad (1.1)$$

such that $\{A_{n,k}\}_{n,k=0}^{\infty}$ is a double bounded sequence of real or complex numbers, $\mathbb{N}$ be a set of non-negative integers, the symbol [a] indicate to the largest integer in $a \in \Re$ and indicate to the Pochhammer symbol $(\lambda)_n$ given by [11]

$$(\lambda)_n = \frac{\Gamma(\lambda + n)}{\Gamma(\lambda)}, \quad \lambda \neq 0, -1, -2, \dots, \qquad (1.2)$$

where $\Gamma(\cdot)$ is Gamma function.

The Srivastava polynomials $S_n^N(x)$ is extended by Gonzalez *et al.* [5] as follows:

$$S_{n,m}^N(x) = \sum_{k=0}^{\left[\frac{n}{N}\right]} \frac{(-n)_{Nk}}{k!} A_{n+m,k}\, x^k \quad (n,m \in \mathbb{N}_0\,; N \in \mathbb{N}), \qquad (1.3)$$

The family of one-variable, two-parameter $S_n^{p,q}(x)$ are defined by [8]:

$$S_n^{p,q}(x) = \sum_{k=0}^{n} \frac{(-n)_k}{k!} A_{p+q+n,q+k}\, x^k \quad (p,q,n,k \in \mathbb{N}_0), \qquad (1.4)$$

Another extension of the Srivastava polynomials is given by Kaanoglu et al. [8] as:

$$S_n^{\rho,\sigma}(w,v) = \sum_{k=0}^{n} A_{\rho+\sigma+n,\sigma+k} \frac{w^k}{k!} \frac{v^{n-k}}{(n-k)!} \quad (\rho, \sigma, n, k \in \mathbb{N}_0), \qquad (1.5)$$

such that $\{A_{n,k}\}$ is a bounded double sequence of any number, real or complex.

In [8], Kaanoglu et al. introduced the three-variable polynomials as follow:

$$S_n^{p,q,M}(x,y,z) = \sum_{k=0}^{n} \sum_{l=0}^{[k/M]} A_{p+q+n,q+k,l} \frac{x^l}{l!} \frac{y^{k-Ml}}{(k-Ml)!} \frac{z^{n-k}}{(n-k)!} \quad (p,q,k,l \in \mathbb{N}_0, M \in \mathbb{N}), \qquad (1.6)$$

where $\{A_{n,k,l}\}$ is a triple sequence of complex numbers. Suitable choices of $\{A_{n,k,l}\}$ in equation (1.6) give a three-variable version of well-known polynomials (see also [6]).

The multivariable extension of Srivastava polynomials in r-variable was recently introduced in [7] as.

$$S_n^{m,N_1,N_2,\ldots,N_{r-1}}(x_1,x_2,\ldots,x_r) := \sum_{k_{r-1}=0}^{\left[\frac{n}{N_{r-1}}\right]} \sum_{k_{r-2}=0}^{\left[\frac{k_{r-1}}{N_{r-2}}\right]} \cdots \sum_{k_2=0}^{\left[\frac{k_3}{N_2}\right]} \sum_{k_1=0}^{\left[\frac{k_2}{N_1}\right]} A_{m+n,k_{r-2},k_1,k_2,\ldots,k_{r-1}}$$

$$\frac{x_1^{k_1}}{k_1!} \frac{x_2^{k_2-N_1k_2}}{(k_2-N_1k_2)!} \cdots \frac{x_r^{n-N_{r-1}k_{r-1}}}{(n-N_{r-1}k_{r-1})!}, \qquad (1.7)$$

$$(m,n \in \mathbb{N}_0;\ N_1,N_1,\ldots,k_{r-1} \in \mathbb{N})$$

Such that $\{A_{m,k_{r-2},k_1,k_2,\ldots,k_{r-1}}\}$ is a sequence of complex numbers.

The two variable Laguerre polynomials (TVLP) are defined by series ([2]; p.!21(69))

$$L_n(x,y) = n! \sum_{k=0}^{n} \frac{(-1)^k\, x^k\, y^{n-k}}{(k!)^2\,(n-k)!}, \qquad (1.8)$$

and specified by the following generating functions:

$$\sum_{n=0}^{\infty} \frac{(c)_n\, L_n(x,y)\, t^n}{n!} = (1-yt)^{-c}\ {}_1F_1\left[c,1, \quad \frac{-xt}{1-yt}\right] \quad |yt| < 1 \qquad (1.9)$$

Also $g_n^{(\alpha_1,\cdots,\alpha_r)}(x_1,\cdots,x_r)$ is Lagrange polynomials of r-variables, given by the following result [1]:

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{(\mu)_n}\, g_n^{(\alpha_1,\cdots,\alpha_r)}(x_1,\cdots,x_r)t^n = F_D^{(r)}[\lambda,\alpha_1,\ldots,\alpha_r\ ;\ \mu;\ x_1t,\ldots,x_rt], \qquad (1.10)$$

where $F_D^{(r)}$ is the Lauricella's function of the fourth kind of several variables defined by [11]

$$F_D^{(r)}\left(a,b_1,\cdots,b_r;c;x_1,\cdots,x_r\right)$$
$$= \sum_{m_1,\cdots,m_r=0}^{\infty} \frac{(a)_{m_1+\cdots+m_r}(b_1)_{m_1}\cdots(b_r)_{m_r}}{(c)_{m_1+\cdots+m_r}} \frac{x_1^{m_1}}{m_1!}\cdots\frac{x_r^{m_r}}{m_r!}, \qquad (1.11)$$
$$\max\{|x_1|,\cdots,|x_r|\}<1.$$

The special case of (1.10) when $r=2$ *and* $\mu=1$ gives the following result:

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{n!}\, g_n^{(\delta,\gamma)}(x_1,x_2)\, t^n = F_1[\lambda,\delta,\gamma;1;x_1t,x_2t] \qquad (1.12)$$

where F_1 is Appell double hypergeometric functions [11]

$$F_1[a,b_1,b_2\ ;\ c\ ;x\ y] = \sum_{m,n=0}^{\infty} \frac{(a)_{m+n}\,(b_1)_{m+n}\,(b_2)_{m+n}}{(c)_{m+n}} \frac{x^m}{m!} \frac{y^n}{n!}. \qquad (1.13)$$

Kaanoglu et al. [8] provided a definition of two-variable polynomials $P_{p,q}^{M}(x,y)$ as follows:

$$P^{M}_{m_1,m_2}(x,y)=\sum_{k=0}^{[m_2/M]} A_{m_1+m_2,m_2,k}\,\frac{x^{m_2-Mk}}{(m_2-Mk)\,!}\,\frac{y^k}{k!}. \qquad (1.14)$$

Note that in (1.14), if we set $M=1$ and $A_{m,n,k}=(\alpha)_k\,(\beta)_{n-k}(\gamma)_{m-n}\ (m,n\in N_0)$, we have

$$P^{1}_{m_1,m_2}(x,y)=(\gamma)_{m_1}\,g^{(\beta,\alpha)}_{m_2}(x,y). \qquad (1.15)$$

Furthermore, choosing $M=2$ and $A_{m,n,k}=(\alpha)_{m-n}(\gamma)_{n-2k}(\beta)_k\ (m,n\in N_0)$ in defined (1.14), then

$$P^{2}_{m_1,m_2}(x,y)=(\alpha)_{m_1}\,h^{(\gamma,\beta)}_{m_2}(x,y), \qquad (1.16)$$

where $g^{(\beta,\alpha)}_{m_2}(x,y)$ denotes the Lagrange polynomials given by

$$g^{(\beta,\alpha)}_{m_2}(x,y)=\sum_{k=0}^{[m_2]}(\alpha)_{m_2-l}\,(\beta)_l\,\frac{x^{m_2-l}}{(m_2-l)\,!}\,\frac{y^l}{l!}, \qquad (1.17)$$

where $h^{(\gamma,\beta)}_{m_2}(x,y)$ denotes the Lagrange-Hermite polynomials given explicitly

$$h^{(\gamma,\beta)}_{m_2}(x,y)=\sum_{l=0}^{[m_2/M]}(\gamma)_{m_2-2l}\,(\beta)_l\,\frac{x^{m_2-2l}}{(m_2-2l)\,!}\,\frac{y^l}{l!}. \qquad (1.18)$$

If we set $M=1$ in (1.6) and $A_{m,n,k}=(\alpha)_k\,(\beta)_{n-k}\,(\gamma)_{m-n}$, we get the following result:

$$S^{p,q,1}_{n}(x,y,z)=(\gamma)_p(\beta)_q\,g^{(\alpha,\beta+q,\gamma+p)}_{n}(x,y,z). \qquad (1.19)$$

Also If we set $M=2$ in (1.6) and $A_{m,n,k}=(\alpha)_{m-n}(\gamma)_{n-2k}(\beta)_k$, we get the following result:

$$S^{p,q,2}_{n}(x,y,z)=(\alpha)_p(\gamma)_q\,u^{(\alpha+p,\beta,\gamma+q)}_{n}(x,y,z). \qquad (1.20)$$

where $u^{(\alpha,\beta,\gamma)}_{n}(x,y,z)$ is the polynomials given by

$$u^{(\alpha,\beta,\gamma)}_{n}(x,y,z)=\sum_{k=0}^{n}\sum_{l=0}^{[k/2]}(\alpha)_l(\beta)_{k-l}(\gamma)_{n-k}\frac{y^l}{l!}\frac{x^{n-k}}{(n-k)!}\frac{z^{k-2l}}{(k-2l)!} \qquad (1.21)$$

2. Main Results

Theorem 2.1 Lagurre polynomials of two variables and Srivastava polynomials with Two-parameter three-variable satisfied the following result:

$$\sum_{p,q,n=0}^{\infty} L_{p+q+n}(x,y)\,S^{p,q,M}_{n}(u,v,w)\,\frac{{z_1}^p}{p!}\,\frac{{z_2}^q}{q!}\,t^n$$

$$=\sum_{p,q,l=0}^{\infty} L_{p+q+Ml}(x,y)\,A_{p+q+Ml,q+Ml,l}\,\frac{(ut^M)^l}{l!}\,\frac{(z_1+wt)^p}{p!}\,\frac{(z_2+vt)^q}{q!}. \qquad (2.1)$$

Proof: Let Δ symbolize of the left hand side of (2.1) and $S^{p,q,M}_{n}(u,v,w)$ expresses the equation (1.6) :

$$\Delta=\sum_{p,q,n=0}^{\infty} L_{p+q+n}(x,y)\sum_{k=0}^{n}\sum_{l=0}^{[k/M]} A_{p+q+n,q+k,l}\,\frac{u^l}{l!}\,\frac{v^{k-Ml}}{(k-Ml)!}\,\frac{w^{n-k}}{(n-k)!}\,\frac{{z_1}^p}{p!}\,\frac{{z_2}^q}{q!}\,t^n,$$

Let $n\to n+k$

$$\Delta=\sum_{p,q,n,k=0}^{\infty} L_{p+q+n+k}(x,y)\sum_{l=0}^{[k/M]} A_{p+q+n+k,q+k,l}\,\frac{u^l}{l!}\,\frac{v^{k-Ml}}{(k-Ml)!}\,\frac{w^n}{n!}\,\frac{{z_1}^p}{p!}\,\frac{{z_2}^q}{q!}\,t^{n+k}$$

Let $k\to k+Ml$

$$\Delta=\sum_{p,q,n,k,l=0}^{\infty} L_{p+q+n+k+Ml}(x,y)\,A_{p+q+n+k+Ml,q+k+Ml,l}\,\frac{(ut^M)^l}{l!}\,\frac{(vt)^k}{k!}\,\frac{(wt)^n}{n!}\,\frac{{z_1}^p}{p!}\,\frac{{z_2}^q}{q!}$$

Let $p\to p-n$

$$\Delta = \sum_{p,q,k,l=0}^{\infty} L_{p+q+k+Ml}(x,y)\, A_{p+q+k+Ml,q+k+Ml,l}\, \frac{(ut^M)^l}{l!}\, \frac{(vt)^k}{k!} \frac{z_2{}^q}{q!} \left(\sum_{n=0}^{p} \frac{z_1{}^{p-n}}{(p-n)!}\, \frac{(wt)^n}{n!} \right)$$

$$\Delta = \sum_{p,q,k,l=0}^{\infty} L_{p+q+k+Ml}(x,y)\, A_{p+q+k+Ml,q+k+Ml,l}\, \frac{(z_1+wt)^p}{p!}\, \frac{(ut^M)^l}{l!}\, \frac{(vt)^k}{k!} \frac{z_2{}^q}{q!}$$

Let $q \to q - k$

$$\Delta = \sum_{p,q,l=0}^{\infty} L_{p+q+Ml}(x,y)\, A_{p+q+Ml,q+Ml,l}\, \frac{(z_1+wt)^p}{p!}\, \frac{(ut^M)^l}{l!} \left(\sum_{k=0}^{q} \frac{(vt)^k}{k!}\, \frac{z_2{}^{q-k}}{(q-k)!} \right)$$

$$\Delta = \sum_{p,q,l=0}^{\infty} L_{p+q+Ml}(x,y)\, A_{p+q+Ml,q+Ml,l}\, \frac{(ut^M)^l}{l!}\, \frac{(z_1+wt)^p}{p!}\, \frac{(z_2+vt)^q}{q!}.$$

Therefore, the equation (2.1) holds.

Similarly, we right away obtain the following result.

Theorem 2.2 The following bilateral generating function family is true:

$$\sum_{p,q,n=0}^{\infty} g_{p+q+n}^{(\gamma,\delta)}(x,y)\, S_n^{p,q,M}(u,v,w)\, \frac{z_1{}^p}{p!}\, \frac{z_2{}^q}{q!}\, t^n$$

$$= \sum_{p,q,l=0}^{\infty} g_{p+q+Ml}^{(\gamma,\delta)}(x,y)\, A_{p+q+Ml,q+Ml,l}\, \frac{(ut^M)^l}{l!}\, \frac{(z_1+wt)^p}{p!}\, \frac{(z_2+vt)^q}{q!}. \qquad (2.2)$$

Note that, if we let $q \to q - Ml$ in the r.h.s. of (2.1) and (2.2) and then using (1.14), we get:

$$\sum_{p,q,n=0}^{\infty} L_{p+q+n}(x,y)\, S_n^{p,q,M}(u,v,w)\, \frac{z_1{}^p}{p!}\, \frac{z_2{}^q}{q!}\, t^n$$

$$= \sum_{p,q,l=0}^{\infty} L_{p+q}(x,y)\, \frac{(z_1+wt)^p}{p!}\, P_{p,q}^M(z_2+vt, ut^M), \qquad (2.3)$$

$$\sum_{p,q,n=0}^{\infty} g_{p+q+n}^{(\gamma,\delta)}(x,y)\, S_n^{p,q,M}(u,v,w)\, \frac{z_1{}^p}{p!}\, \frac{z_2{}^q}{q!}\, t^n$$

$$= \sum_{p,q,l=0}^{\infty} g_{p+q}^{(\gamma,\delta)}(x,y)\, \frac{(z_1+wt)^p}{p!}\, P_{p,q}^M(z_2+vt, ut^M). \qquad (2.4)$$

Now, using (1.15), (1.19) in (2.3) and (2.4) respectively and then using (1.16) and (1.20), we have

$$\sum_{p,q,n=0}^{\infty} L_{p+q+n}(x,y)\, (\gamma)_p (\beta)_q\, g_n^{(\alpha,\beta+q,\gamma+p)}(u,v,w)\, \frac{z_1{}^p}{p!}\, \frac{z_2{}^q}{q!}\, t^n$$

$$= \sum_{p,q=0}^{\infty} L_{p+q}(x,y)\, \frac{(z_1+wt)^p}{p!}\, P_{p,q}^1(z_2+vt, ut^1), \qquad (2.5)$$

$$\sum_{p,q,n=0}^{\infty} g_{p+q+n}^{(\gamma,\delta)}(x,y)\, (\gamma)_p (\beta)_q\, g_n^{(\alpha,\beta+q,\gamma+p)}(u,v,w)\, \frac{z_1{}^p}{p!}\, \frac{z_2{}^q}{q!}\, t^n$$

$$= \sum_{p,q=0}^{\infty} g_{p+q}^{(\gamma,\delta)}(x,y) \frac{(z_1+wt)^p}{p!} P_{p,q}^1(z_2+vt, ut^1), \qquad (2.6)$$

and

$$\sum_{p,q,n=0}^{\infty} L_{p+q+n}(x,y) (\alpha)_p (\gamma)_q u_n^{(\alpha+p,\beta,\gamma+q)}(u,v,w) \frac{z_1{}^p}{p!} \frac{z_2{}^q}{q!} t^n$$

$$= \sum_{p,q=0}^{\infty} L_{p+q}(x,y) \frac{(z_1+wt)^p}{p!} P_{p,q}^2(z_2+vt, ut^2), \qquad (2.7)$$

$$\sum_{p,q,n=0}^{\infty} g_{p+q+n}^{(\gamma,\delta)}(x,y) (\alpha)_p (\gamma)_q u_n^{(\alpha+p,\beta,\gamma+q)}(u,v,w) \frac{z_1{}^p}{p!} \frac{z_2{}^q}{q!} t^n$$

$$= \sum_{p,q=0}^{\infty} g_{p+q}^{(\gamma,\delta)}(x,y) \frac{(z_1+wt)^p}{p!} P_{p,q}^2(z_2+vt, ut^2). \qquad (2.8)$$

Using (1.15) in (2.5), (2.6) and using (1.16) in (2.7), (2.8), we have

$$\sum_{p,q,n=0}^{\infty} L_{p+q+n}(x,y) (\gamma)_p (\beta)_q g_n^{(\alpha,\beta+q,\gamma+p)}(u,v,w) \frac{z_1{}^p}{p!} \frac{z_2{}^q}{q!} t^n$$

$$= \sum_{p,q,l=0}^{\infty} L_{p+q}(x,y) \frac{(z_1+wt)^p}{p!} (\gamma)_p g_q^{(\beta,\alpha)}(z_2+vt, ut), \qquad (2.9)$$

$$\sum_{p,q,n=0}^{\infty} g_{p+q+n}^{(\gamma,\delta)}(x,y) (\gamma)_p (\beta)_q g_n^{(\alpha,\beta+q,\gamma+p)}(u,v,w) \frac{z_1{}^p}{p!} \frac{z_2{}^q}{q!} t^n$$

$$= \sum_{p,q=0}^{\infty} g_{p+q}^{(\gamma,\delta)}(x,y) \frac{(z_1+wt)^p}{p!} (\gamma)_p g_q^{(\beta,\alpha)}(z_2+vt, ut), \qquad (2.10)$$

and

$$\sum_{p,q,n=0}^{\infty} L_{p+q+n}(x,y) (\alpha)_p (\gamma)_q u_n^{(\alpha+p,\beta,\gamma+q)}(u,v,w) \frac{z_1{}^p}{p!} \frac{z_2{}^q}{q!} t^n$$

$$= \sum_{p,q=0}^{\infty} L_{p+q}(x,y) \frac{(z_1+wt)^p}{p!} (\alpha)_p h_q^{(\gamma,\beta)}(z_2+vt, ut^2), \qquad (2.11)$$

$$\sum_{p,q,n=0}^{\infty} g_{p+q+n}^{(\gamma,\delta)}(x,y) (\alpha)_p (\gamma)_q u_n^{(\alpha+p,\beta,\gamma+q)}(u,v,w) \frac{z_1{}^p}{p!} \frac{z_2{}^q}{q!} t^n$$

$$= \sum_{p,q=0}^{\infty} g_{p+q}^{(\gamma,\delta)}(x,y) \frac{(z_1+wt)^p}{p!} (\alpha)_p h_q^{(\gamma,\beta)}(z_2+vt, ut^2). \qquad (2.12)$$

Remark 2.1 Choosing $z_1 = -wt$ and $z_2 = -vt$ in (2.1) and (2.2) , we deduce the following interesting corollaries:

Corollary 2.1.

$$\sum_{p,q,n=0}^{\infty} L_{p+q+n}(x,y)\, S_n^{p,q,M}(u,v,w)\, \frac{(-wt)^p}{p!}\, \frac{(-vt)^q}{q!}\, t^n$$
$$= \sum_{l=0}^{\infty} L_{Ml}(u,v)\, A_{Ml,Ml,l}\, \frac{(ut^M)^l}{l!}, \qquad (2.13)$$

Corollary 2.2.

$$\sum_{p,q,n=0}^{\infty} g_{p+q+n}^{(\gamma,\delta)}(x,y)\, S_n^{p,q,M}(u,v,w)\, \frac{(-wt)^p}{p!}\, \frac{(-vt)^q}{q!}\, t^n$$
$$= \sum_{l=0}^{\infty} g_{Ml}^{(\gamma,\delta)}(x,y)\, A_{Ml,Ml,l}\, \frac{(ut^M)^l}{l!} \qquad (2.14)$$

Remark 2.2 Choosing $M = 1, 2$ in (2.13), we get the following result:

$$\sum_{p,q,n=0}^{\infty} L_{p+q+n}(x,y)\, S_n^{p,q,1}(u,v,w)\, \frac{(-wt)^p}{p!}\, \frac{(-vt)^q}{q!}\, t^n$$
$$= \sum_{l=0}^{\infty} L_l(x,y)\, A_{l,l,l}\, \frac{(ut)^l}{l!}, \qquad (2.15)$$

and

$$\sum_{p,q,n=0}^{\infty} L_{p+q+n}(x,y)\, S_n^{p,q,2}(u,v,w)\, \frac{(-wt)^p}{p!}\, \frac{(-vt)^q}{q!}\, t^n$$
$$= \sum_{l=0}^{\infty} L_{2l}(x,y)\, A_{2l,2l,l}\, \frac{(ut^2)^l}{l!}. \qquad (2.16)$$

Remark 2.3 Choosing $M = 1, 2$ in (2.14), we get the following result:

$$\sum_{p,q,n=0}^{\infty} g_{p+q+n}^{(\gamma,\delta)}(x,y)\, S_n^{p,q,1}(u,v,w)\, \frac{(-wt)^p}{p!}\, \frac{(-vt)^q}{q!}\, t^n$$
$$= \sum_{l=0}^{\infty} g_l^{(\gamma,\delta)}(x,y)\, A_{l,l,l}\, \frac{(ut)^l}{l!}, \qquad (2.17)$$

and

$$\sum_{p,q,n=0}^{\infty} g_{p+q+n}^{(\gamma,\delta)}(x,y)\, S_n^{p,q,2}(u,v,w)\, \frac{(-wt)^p}{p!}\, \frac{(-vt)^q}{q!}\, t^n$$
$$= \sum_{l=0}^{\infty} g_{2l}^{(\gamma,\delta)}(x,y)\, A_{2l,2l,l}\, \frac{(ut^2)^l}{l!}. \qquad (2.18)$$

3. Applications

I. In (2.15) and (2.17), choosing $A_{l,l,l} = (\alpha)_l$ and using (1.19), we get:

$$\sum_{p,q,n=0}^{\infty} L_{p+q+n}(x,y)\,(\gamma)_p(\beta)_q\, g_n^{(\alpha,\beta+q,\gamma+p)}(u,v,w)\,\frac{(-wt)^p}{p!}\,\frac{(-vt)^q}{q!}\,t^n$$

$$=\sum_{l=0}^{\infty}(\alpha)_l\, L_l(x,y)\,\frac{(ut)^l}{l!}, \qquad (3.1)$$

and

$$\sum_{p,q,n=0}^{\infty} g_{p+q+n}^{(\gamma,\delta)}(x,y)\,(\gamma)_p(\beta)_q\, g_n^{(\alpha,\beta+q,\gamma+p)}(u,v,w)\,\frac{(-wt)^p}{p!}\,\frac{(-vt)^q}{q!}\,t^n$$

$$=\sum_{l=0}^{\infty}(\alpha)_l\, g_l^{(\gamma,\delta)}(x,y)\,\frac{(ut)^l}{l!}. \qquad (3.2)$$

Using relation (1.9) in the L. H. S. of result (3.1), we get:

$$\sum_{p,q,n=0}^{\infty} L_{p+q+n}(x,y)\,(\alpha)_p(\beta)_q\, g_n^{(\alpha,\beta+q,\gamma+p)}(u,v,w)\,\frac{(-wt)^p}{p!}\,\frac{(-vt)^q}{q!}\,t^n$$

$$=(1-yut)^{-\alpha}\ {}_1F_1\left[\alpha,1,\frac{-xut}{1-yvt}\right]\ |yvt|<1. \qquad (3.3)$$

and using relation (1.10) in the L. H. S. of results (3.2), we get:

$$\sum_{p,q,n=0}^{\infty} g_{p+q+n}(x,y)\,(\alpha)_p(\beta)_q\, g_n^{(\alpha,\beta+q,\gamma+p)}(u,v,w)\,\frac{(-wt)^p}{p!}\,\frac{(-vt)^q}{q!}\,t^n$$

$$=F_1[\alpha,\gamma,\delta;1;xut,yut]. \qquad (3.5)$$

II. In (2.16) and (2.18), choosing $A_{2l,2l,l}=(\beta)_l$ and using (1.20), we get

$$\sum_{p,q,n=0}^{\infty} L_{p+q+n}(x,y)\,(\alpha)_p(\gamma)_q\, u_n^{(\alpha+p,\beta,\gamma+q)}(u,v,w)\,\frac{(-wt)^p}{p!}\,\frac{(-vt)^q}{q!}\,t^n$$

$$=\sum_{l=0}^{\infty}(\beta)_l\, L_{2l}(x,y)\,\frac{(ut^2)^l}{l!}, \qquad (3.6)$$

and

$$\sum_{p,q,n=0}^{\infty} g_{p+q+n}^{(\gamma,\delta)}(x,y)\,(\alpha)_p(\gamma)_q\, u_n^{(\alpha+p,\beta,\gamma+q)}(u,v,w)\,\frac{(-wt)^p}{p!}\,\frac{(-vt)^q}{q!}\,t^n$$

$$=\sum_{l=0}^{\infty}(\beta)_l\, g_{2l}^{(\gamma,\delta)}(x,y)\,\frac{(ut^2)^l}{l!}. \qquad (3.7)$$

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conficts of interest.

How to Cite : Salem Saleh Barahmah (2023).Certain results for the three- variable Srivastava polynomials with two parameters, *Abhath Journal of Basic and Applied Sciences*, 2(2), 21-28.

References.

[1] Chan, W.-C.C., Chyan, C.-J., Srivastava, H.M.: The Lagrange polynomials in Several variables, Integral Transforms Spec. Funct., **12**:139-1412 (2001).

[2] Dattoli, G.: Generalized polynomials, operational identities and their applications. Journal of Computational andApplied Mathematics, **1112**, 111–123(2000).

[3] Dattoli , G., Torre, A.: Operational methods and two variable Laguerre polynomials. Atti Accad. Sci. TorinoCl. Sci. Fis. Mat. Natur., **132**, 1–7 (19912).

[4] Dattoli, G., Torre, A., Mancho, A. M.: The generalized Laguerre polynomials, the associated Bessel functionsand applications to propagation problems. Radiat Phys Chem., **59**, 229–237(2000).

[5] Gonzalez, B., Matera, J., Srivastava, H. M.: Some q-generating functions and associated generalized hypergeometric polynomials, Math. Comput. Mod., **34**, 133-175 (2001).

[6] Kaanoglu , C., Özarslan, M.A.: New families of generating functions for certain class of three-variable polynomials. Appl. Math. Comput., **2112**, 1236-1242 (2011).

[7] Kaanoglu , C., Özarslan, M.A.: Two-sided generating functions for certain class of r-variable polynomials. Math. Comput. Model, **54**, 625-631 (2011).

[8] Kaanoglu , C., Özarslan, M.A.: Two-parameter Srivastava polynomials and several series identities, Adva. Diff. Equa., **121**, 1-9 (2013).

[9] Rainville, E.D.: Special Functions, Macmillan, New York, 1960, reprinted by Chelsea Publ. Co., Bronx, New York, 1971.

[10] Srivastava, H. M.: A contour integral involving Fox's H-function, Indian J. Math., 14, 1-6 (1972).

[11] Srivastava , H. M., Manocha, H. L.: A Treatise on Generating Functions, Halsted Press, New York, (1984).