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Vaccination-based Measles Outbreak Model with Fractional Dynamics

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Abstract

The objective of this study is to create and evaluate a novel measles model that takes into account the impact of vaccination in Yemen and makes use of fractional piecewise Caputo derivatives. The theoretical aspect provides the disease-free equilibrium (DFE) points, the basic reproduction number (R0), and the biologically viable region of the proposed model. We also deduce the results for uniqueness using the Banach fixed point theorem.

Keywords: *Measles , Outbreak , Mathematical Model , Fractional Dynamic, fixed point theorem.*

1. Introduction

Measles is a highly contagious, serious disease caused by a virus. Before the introduction of measles vaccine in 1963 and widespread vaccination, major epidemics occurred approximately every 2–3 years and measles caused an estimated 2.6 million deaths each year [1]. Measles is an acute viral respiratory illness. It is characterized by a prodrome of fever (as high as 105°F) and malaise, cough, coryza, and conjunctivitis -the three "C"s -, a pathognomonic enanthema (Koplik spots) followed by a maculopapular rash. The rash usually appears about 14 days after a person is exposed. The rash spreads from the head to the trunk to the lower extremities. Patients are considered to be contagious from 4 days before to 4 days after the rash appears [2]. Measles remains an important cause of child morbidity and mortality worldwide despite the availability of a safe and efficacious vaccine [3]. The current measles virus vaccine was developed empirically by attenuation of wild-type measles virus vaccine by in vitro passage in human and chicken cells and licensed in 1963. Additional passages led to further attenuation and the successful vaccine strains in widespread use today [4]. For measles, the basic reproduction number (R_0) is often cited to be 12-18, which means that each person with measles would, on average, infect 12-18 other people in a totally susceptible population [5]. $R_0$is defined as the average number of secondary cases of an infectious disease arising from a typical case in a totally susceptible population [6], therefore , the R_0can be estimated in populations if pre-existing immunity can be accounted for in the calculation based on mathematical model . The SIR model has been modified to adapt for measles with an incubation period [7,8]. Bakare [7] explained the characteristics of measles in 2012 using the model below

$$\begin{cases} S'(t) = \lambda - \sigma S I \frac{1}{N} - \kappa S, \\ E'(t) = \sigma S I \frac{1}{N} - (\eta + \kappa) E, \\ I'(t) = \eta E - (\gamma + \kappa) E, \\ R' = \gamma I - \kappa R, \end{cases} \tag{1.1}$$

The accompanying model for the model (1.1) is

$$\begin{cases} S'(t) = m(1-p)N - \sigma S \frac{1}{N} - \kappa S, \\ E'(t) = \sigma S \frac{1}{N} - (\eta + \kappa) E, \\ I'(t) = \eta E - (\gamma + \zeta + \kappa) E, \\ R' = mpN + \gamma I - \kappa R, \end{cases} \tag{2.1}$$

which has been studied by Tessa [9].

In the past, researchers have modeled real-world issues using local operators and classical differential equations [10]. However, it is challenging to show how memory and genetic traits impact a wide range of processes and events.

Experts were therefore interested to investigate these problems in terms of FDEs, which have lately gained attention due to their obvious novelty. A number of definitions and techniques have been proposed to describe the behavior of some challenging real-world problems that emerge in a variety of scientific fields as a result of FC becoming a rich source of information for experts, see

[11-13]. The multi-step behavior that some situations display is one of these issues. The concept of the multi-definition derivative was introduced by Atangana and Araz [14].
The following piecewise fractional model of vaccination-based measles is taken into consideration by combining the two models (1.1) and (2.1) in two finite periods.

$$\begin{cases} {}^{PCC}D_0^{\alpha}\, S(t) = m(1-p)N - \sigma S\frac{I}{N} - \kappa S, \\ {}^{PCC}D_0^{\alpha}\, E(t) = \sigma S\frac{I}{N} - (\eta+\kappa)E, \\ {}^{PCC}D_0^{\alpha}\, I(t) = \eta E - (\gamma+\zeta+\kappa)I, \\ {}^{PCC}D_0^{\alpha}\, R(t) = mpN + \gamma I - \kappa R, \end{cases} \tag{3.1}$$

with initial conditions

$$S(0) = S_0, E(0) = I_0, I(0) = V_0,\ R(0) = R_0$$

where $0 < \alpha \leq 1,$ ${}^{PCC}D_0^{\alpha}$ stands for classical derivative in $0 \leq t \leq t_1$ and represents Caputo fractional derivative in $t_1 \leq t \leq \mho < \infty,$ and $S, E, I,$ and R as susceptible, exposed, infectious and the immune individuals and N is population. The parameters are defined as table following:

Table 1. Biological description of model parameters and their numerical values

Parameter	Description	Values
ζ	Differential mortality due to measles	0.13
$\frac{1}{\eta}$	Average latent period	$\frac{1}{2.6}$
$\frac{1}{\gamma}$	Average infectious period	$\frac{1}{3.2}$
σ	Contact rate	18
m	Birth rate	6.2
p	Proportion of those successively vaccinated at birth	0.80
κ	Mortality rate	2.52

The model (1.3) can be expressed as

$$\begin{cases} {}^{PCC}D_0^{\alpha}S(t) = \begin{cases} \frac{d}{dt}S(t) = F_1(t,S,E,I,R),\ 0 \leq t \leq t_1 \\ {}^{C}D_{t_1}^{\alpha}S(t) = F_1(t,S,E,I,R),\ t_1 < t \leq \mho, \end{cases} \\ {}^{PCC}D_0^{\alpha}E(t) = \begin{cases} \frac{d}{dt}E(t) = F_2(t,S,E,I,R),\ 0 \leq t \leq t_1, \\ {}^{C}D_{t_1}^{\alpha}E(t) = F_2(t,S,E,I,R),\ t_1 < t \leq \mho, \end{cases} \\ {}^{PCC}D_0^{\alpha}I(t) = \begin{cases} \frac{d}{dt}I(t) = F_3(t,S,E,I,R),\ 0 \leq t \leq t_1, \\ {}^{C}D_{t_1}^{\alpha}I(t) = F_3(t,S,E,I,R),\ t_1 < t \leq \mho, \end{cases} \\ {}^{PCC}D_0^{\alpha}R(t) = \begin{cases} \frac{d}{dt}R(t) = F_4(t,S,E,I,R),\ 0 \leq t \leq t_1, \\ {}^{C}D_{t_1}^{\alpha}R(t) = F_4(t,S,E,I,R),\ t_1 < t \leq \mho, \end{cases} \end{cases}$$

where

$$\begin{cases} F_1(t,S,E,I,R) = m(1-p)N - \sigma S\frac{I}{N} - \kappa S, \\ F_2(t,S,E,I,R) = \sigma S\frac{I}{N} - (\eta+\kappa)E, \\ F_3(t,S,E,I,R) = \eta E - (\gamma+\zeta+\kappa)E, \\ F_4(t,S,E,I,R) = mpN + \gamma I - \kappa R, \end{cases}$$

2. Preliminary results

We start off this part by providing some notations and the fundamental nomenclature. Let $0 \leq t_1 < t \leq \mho < \infty$ and define the Banach space by $\mathbf{X} = C([0,\mho] \times \mathbb{R}^3, \mathbb{R})$ under the norm

$$\|\Phi\| = \max_{t\in[0,\mho]} |\Phi(t)|.$$

Definition *2.1* [14] *Let* Φ *be differentiable. Then the piecewise integration is defined by*

$${}^{PC}I_0^{\theta}\Phi(t) = \begin{cases} \int_0^{t_1} \Phi(\xi)d\xi,\ 0 \leq t \leq t_1, \\ \frac{1}{\Gamma(\theta)}\int_{t_1}^{t} (t-\xi)^{\theta-1}\Phi(\xi)d(\xi),\ t_1 < t \leq \mho, \end{cases}$$

where ${}^{PC}I_0^{\theta}$ *denotes for classical integral in* $0 \leq t \leq t_1$ *and represent Riemann–Liouville fractional integral in* $t_1 < t \leq \mho.$

Definition *2.2* [14] *Let* $\Phi \in C[0,\mho]$ *be differentiable. Then the piecewise derivative is given as*

$${}^{PCC}D_0^{\theta}\Phi(t) = \begin{cases} \Phi'(t), & 0 \leq t \leq t_1, \\ {}^{C}D_{t_1}^{\theta}\Phi(t), & t_1 < t \leq \mho. \end{cases}$$

where ${}^{PCC}D_0^{\theta}$ *denotes for classical derivative in* $0 \leq t \leq t_1$ *and represent Caputo fractional derivative in* $t_1 < t \leq \mho.$

Lemma 2.3 [14]. *The solution of the piecewise problem*

$${}^{PCC}D_0^{\theta}\Phi(t) = g(t),\ 0 < \theta \leq 1$$

is given by

$$\Phi(t) = \begin{cases} \Phi_0 + \int_0^t g(\xi)d\xi, \ 0 \le t \le t_1, \\ \Phi(t_1) + \frac{1}{\Gamma(\theta)} \int_{t_1}^t (t-\xi)^{\theta-1} g(\xi)d(\xi), \ t_1 < t \le \mho. \end{cases}$$

3. Main Results

It has been perceived that in many measles models the system attains its measles-free state when the vaccine appears in the population. We now give the existence condition for the nonnegative solution of the model (1.3).

Let $\mathbb{R}_+^4 = \{\Phi \in \mathbb{R}^4 : \Phi(t) \ge 0\}$ and let $\Phi(t) = (S(t), E(t), I(t), R(t))^T$. To prove the non-negative solution of model (1.3), we shall need the next generalized mean value theorem

Theorem 3.1 *[9] Let* $h(t) \in C[a, \mho]$ *and*

${}^C D_0^\alpha \in C(a, \mho]$ *for* $0 < \alpha \le 1$. *Then for* $t \in [a, \mho]$ *, we have*

$$\frac{h(t) - h(a)}{t - a} = \frac{1}{\Gamma(\alpha)} {}^C D_0^\alpha h(c), \ c \in [a, t].$$

Corollary 3.2

- *Let* ${}^C D_0^\alpha h(t) \ge 0$, *for all* $t \in (a, \mho)$. *Then* h *is nondecreasing on* $[a, \mho]$.
- *Let* ${}^C D_0^\alpha h(t) \ge 0$, *for all* $t \in (a, \mho)$. *Then* h *is nonincreasing on* $[a, \mho]$.

Theorem 3.3 *A unique solution*

$\Phi(t) = (S(t), E(t), I(t), R(t))^T$ *for* $t > 0$ *of the model (1.3) exists and will remains in* $\mathbb{R}_+^4$. *Moreover,* Φ *is definite positive.*

Proof: *We consider the human population model, given by the four systems of equations. Hence, it is sufficient to consider the dynamics of the human system in*

$\Delta = \{\Phi \in \mathbb{R}_+^4 : \Phi(t) \ge 0\}$. *The solution*

$\Phi(t) = (S(t), E(t), I(t), R(t))^T$ *with* $\Phi(0) = (S(0), E(0), I(0), R(0))^T$ *exists and is unique on interval* $[0, \infty)$. *Then*

$$\begin{cases} {}^{PCC} D_0^\alpha \ S(t)\big|_{S=0} = mN \ge 0 \\ {}^{PCC} D_0^\alpha \ E(t)\big|_{E=0} = \sigma S \frac{I}{N} \ge 0, \\ {}^{PCC} D_0^\alpha \ I(t)\big|_{I=0} = \eta E \ge 0, \\ {}^{PCC} D_0^\alpha \ R(t)\big|_{R=0} = mpN + \gamma I \ge 0, \end{cases}$$

As per Corollary 3.2, we deduced that the solution will be in $\mathbb{R}_+^4$ for all for $t > 0$.

Next, to prove the system's feasible region of (1.3) is bounded we have $N(t) = S(t) + E(t) + I(t) + R(t)$, then by applying the piecewise Caputo operator along with using model (1.3), we get

$$\begin{aligned} {}^{PCC} D_0^\alpha \ N &= mN - \kappa S - \kappa E - \kappa I - \kappa R - \zeta I \\ {}^{PCC} D_0^\alpha \ N &= (m - \kappa)N - \zeta I \\ {}^{PCC} D_0^\alpha \ N &\le (m - \kappa)N. \end{aligned}$$

It follows that

$$\begin{cases} N'(t) \le (m - \kappa)N, \quad 0 \le t \le t_1, \\ {}^C D_{t_1}^\theta N(t) \le (m - \kappa)N, \ t_1 < t \le \mho. \end{cases}$$

and

$$\begin{cases} N(t) \le e^{(m-\kappa)t}, \quad 0 \le t \le t_1, \\ N(t) \le E_\theta[(m - \kappa)t^\theta], \ t_1 < t \le \mho. \end{cases}$$

where, E_θ is Mittag-Leffler function which has an asymptotic behavior, hence, when $t \to \infty$, we get

$$N(t) \subseteq [0, m - \kappa].$$

3.1 Equilibria Points and Basic Reproduction Number

3.1.1. Disease-Free Equilibrium (DFE)

The model (1.3) has a DFE given by $\mathcal{E}_0 = (S^0, 0, 0, R^0)$. As a result, the next-generation matrix method will be used to investigate the local stability. We calculate the next generation matrix for the systems of equation (1.3) by enumerating the number of ways that

- Fresh infections emerge.
- The variety of ways people can move, but there is only one means to spread an infection.

The Jacobian of (1.3) at the equilibrium point

$$J^*(S^*, E^*, I^*, R^*) = \begin{pmatrix} m(1-p) - \sigma \frac{I^*}{N^*} - \kappa & \sigma \frac{I^*}{N^*} & 0 & mp \\ m(1-p) & -(\eta + \kappa) & \eta & mp \\ m(1-p) - \sigma \frac{S^*}{N^*} & \sigma \frac{S^*}{N^*} & -(\gamma + \zeta + \kappa) & mp + \gamma \\ m(1-p) & 0 & 0 & mp - \kappa \end{pmatrix}$$

In absence of infection $E^* = I^* = 0$, the Jacobian of (1.3) at the disease-free equilibrium is $\mathcal{E}_0 = (S^*, 0, 0, R^*)$

$$J^*(S^*,0,0,R^*) = \begin{pmatrix} m(1-p)-\kappa & 0 & 0 & mp \\ m(1-p) & -(\eta+\kappa) & \eta & mp \\ m(1-p)-\sigma\frac{m(1-p)}{\kappa} & \sigma\frac{m(1-p)}{\kappa} & -(\gamma+\zeta+\kappa) & mp+\gamma \\ m(1-p) & 0 & 0 & mp-\kappa \end{pmatrix}$$

Its eigenvalues are

$$\left|J^*-\lambda I\right| = \begin{vmatrix} m(1-p)-\kappa-\lambda & 0 & 0 & mp \\ m(1-p) & -(\eta+\kappa)-\lambda & \eta & mp \\ m(1-p)-\sigma\frac{m(1-p)}{\kappa} & \sigma\frac{m(1-p)}{\kappa} & -(\gamma+\zeta+\kappa)-\lambda & mp+\gamma \\ m(1-p) & 0 & 0 & mp-\kappa-\lambda \end{vmatrix}$$

$\lambda_1 = -\kappa, \lambda_2 = -(m-\kappa)$ and the roots of $X^2 + (2\kappa+\gamma+\zeta+\eta)X + (\eta+\kappa)(\gamma+\zeta+\kappa) - \frac{\sigma m(1-p)}{\kappa}$.

Theorem 3.1.1. *The disease-free equilibrium*

$\mathcal{E}_0 = (S^*,0,0,R^*)$ *is locally stable if* $R_p < 1$ *and*

unstable if $R_p > 1$ *where*

$$R_p = \frac{(1-p)m\eta\sigma}{\kappa(\eta+\kappa)(\gamma+\zeta+\kappa)} \approx 0.76 < 1.$$

Moreover, if $p = 0,$ *then basic reproductive number is*

$$R_0 = \frac{m\eta\sigma}{\kappa(\eta+\kappa)(\gamma+\zeta+\kappa)} \approx 3.8 > 1.$$

where R_p *is the effective reproduction number in presence of vaccination*.

Proof: As λ_1 and λ_2 are negative, it remains to prove that λ_3 and λ_4, the roots of the quadratic part of that characteristic polynomial of J^* are both negative. We know that, using Routh-Hurwitz theorem, it is the case when

$$\lambda_3 + \lambda_4 < 0 \text{ and } \lambda_3\lambda_4 > 0.$$

As $\lambda_3 + \lambda_4 = -(2\kappa+\gamma+\zeta+\eta) < 0$ is true, we are done from

$\lambda_3\lambda_4 = (\eta+\kappa)(\gamma+\zeta+\kappa) - \sigma\frac{m(1-p)}{\kappa} > 0.$

Moreover, *the model Equation (1.3) admits a unique endemic equilibrium point* $\mathcal{E}_1 = (S_1,E_1,I_1,R_1)^T$ *if and only if* $R_0 > 1.$

Lemma 3.5 *(i) The disease-free equilibrium point* $\mathcal{E}_0 = (S^*,0,0,R^*)^T$ *is locally and globally asymptotically stable if and only if* $R_0 < 1$,

(ii) The unique endemic equilibrium point $\mathcal{E}_1 = (S_1,E_1,I_1,R_1)^T$ *is locally stable whenever* $R_0 > 1$.

Remark 3.6 *Note that, by the values estimated in Table 1, we obtain*

$$R_p = \frac{(1-p)m\eta\sigma}{\kappa(\eta+\kappa)(\gamma+\zeta+\kappa)} = \frac{(1-0.80)(6.2)(18)2.6}{2.52(2.6+2.52)(3.2+0.13+2.52)} \approx 0.76 < 1,$$

and

$$R_0 = \frac{m\eta\sigma}{\kappa(\eta+\kappa)(\gamma+\zeta+\kappa)} = \frac{(1)(6.2)(18)2.6}{2.52(2.6+2.52)(3.2+0.13+2.52)} \approx 3.84 > 1.$$

A model or physical problem must be thoroughly examined to confirm its existence. Several theories and analytical methods can be used to generate this concept. The above-mentioned necessity can be studied using the effective instrument of fixed point theory. As a result, In this part, we will examine if the proposed piecewise derivable problem has a unique solution, by using Banach's fixed point theorem. To do this, we write model (1.3) as follows:

$$\begin{cases} {}^{PCC}D_0^{\theta}\Phi(t) = F(t,\Phi(t)), \ 0 < \theta \le 1, \\ \Phi(0) = \Phi_0, \end{cases}$$

where

$$\Phi(t) = \begin{pmatrix} S(t) \\ E(t) \\ I(t) \\ R(t) \end{pmatrix}, \ \Phi(0) = \begin{pmatrix} S(0) \\ E(0) \\ I(0) \\ R(0) \end{pmatrix}, \ \Phi(t_1) = \begin{pmatrix} S(t_1) \\ E(t_1) \\ I(t_1) \\ R(t_1) \end{pmatrix},$$

$$F(t,\Phi(t)) = \begin{pmatrix} F_1(t,S,E,I,R) \\ F_2(t,S,E,I,R) \\ F_3(t,S,E,I,R) \\ F_4(t,S,E,I,R) \end{pmatrix}.$$

The system (1.3) can be transformed to the given integral equation as a result of Definition 2.1 and Lemma 2.3,

$$\Phi(t) = \begin{cases} \Phi(0) + \int_0^t F(\xi,\Phi(\xi))d\xi, \ 0 \le t \le t_1, \\ \Phi(t_1) + \frac{1}{\Gamma(\theta)}\int_{t_1}^t (t-\xi)^{\theta-1}F(\xi,\Phi(\xi))d\xi, \ t_1 < t \le \mho. \end{cases}$$

3.2 Uniquness result

Theorem 3.2.1 *Problem (1.3) has a unique solution if* F *is continuous function and there exists* $L_\Phi > 0$ *such that* $|F(\xi,\Phi_1) - F(\xi,\Phi_2)| \le L_\Phi|\Phi_1 - \Phi_2|,$ *for* $\xi \in \mho, \ \Phi_1,\Phi_2 \in \mathbf{X}$ *, and conditions*

$$\begin{cases} \lambda_1 \coloneqq L_\Phi t_1 < 1, \\ \lambda_2 \coloneqq L_\Phi \frac{(\mho - t_1)^\theta}{\Gamma(\theta+1)} < 1 \end{cases} \tag{3.2.1}$$

are satisfied. As a result, model (1.3) has a unique solution.

Proof: *Let* $\mathcal{O} : \mathbf{X} \to \mathbf{X}$ *be an operator define by (p3). Let* $\Phi, \overline{\Phi} \in \mathbf{X}$, *the*

$$\begin{aligned} \|\mathcal{O}\Phi - \mathcal{O}\overline{\Phi}\| &= \sup_{t\in\mho} |\mathcal{O}\Phi(t) - \mathcal{O}\overline{\Phi}(t)| \\ &\leq \begin{cases} \int_0^t \sup_{t\in[0,t_1]} |F(\xi,\Phi(\xi)) - F(\xi,\overline{\Phi}(\xi))| d\xi, & 0 \leq t \leq t_1, \\ \frac{1}{\Gamma(\theta)} \int_{t_1}^t (t-\xi)^{\theta-1} \sup_{t\in[t_1,\mho]} |F(\xi,\Phi(\xi)) - F(\xi,\overline{\Phi}(\xi))| d\xi, & t_1 < t \leq \mho. \end{cases} \\ &\leq \begin{cases} \|\Phi - \overline{\Phi}\| L_\Phi t, & 0 \leq t \leq t_1, \\ \|\Phi - \overline{\Phi}\| L_\Phi \frac{(t-t_1)^\theta}{\Gamma(\theta+1)}, & t_1 < t \leq \mho. \end{cases} \\ &\leq \begin{cases} \|\Phi - \overline{\Phi}\| L_\Phi t_1, \\ \|\Phi - \overline{\Phi}\| L_\Phi \frac{(\mho - t_1)^\theta}{\Gamma(\theta+1)} \end{cases}. \end{aligned}$$

By (3.2.1), we obtain

$$\|\mathcal{O}\Phi - \mathcal{O}\overline{\Phi}\| \leq \begin{cases} \lambda_1 \|\Phi - \overline{\Phi}\|, \text{ on } [0, t_1] \\ \lambda_2 \|\Phi - \overline{\Phi}\| \text{ on } [t_1, \mho] \end{cases}.$$

Hence the result received.

4. Conclusions

Measles is a highly contagious disease that spreads readily through direct contact or from any transmissible media to susceptible populations. In this article, we discussed a novel measles model that takes into account the impact of vaccination in Yemen and makes use of fractional piecewise Caputo derivatives. The disease-free equilibrium (DFE) points, the basic reproduction number (R_0), and the biologically viable region of the proposed model have been provided. Moreover, we have also deduced the unique solution of the model using the Banach fixed point theorem. This model demonstrates that the vaccination program is the most effective method to stop the spread of measles. Backward bifurcation suggests the potential for inaccurate predictions based on a knowledge of basic reproduction numbers, as measles may still manifest even when R_0 is already less than one. Despite this, we concluded in our paper that R_0 is greater than one. Although the introduction of numerous models since then, there are still some crucial issues that need to be taken into account for future studies, such as the impact of the IgG antibodies that enable the immunity to be passed down from mothers. This immunity only lasts for nine months after the baby is delivered, and it is temporary. The co-infection of measles with other serious diseases, such as pneumonia and diarrhea, is another crucial truth. The model will reveal the complexity of transmission, but an additional contemporary mathematical analysis is required to glean more details.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest**.**

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This work was conducted during our work at Hodeidah University.

Ethical Approval

The studies involving human participants were reviewed and approved by Ethics Committee of CTMES – HU and CTMID, Al-Thawara Public Hospital Authority, Hodeidah, Yemen.

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References

[1] World Health Organizations (WHO) ,(2023), Measles.

[2] Centers for Diseases Control and Prevention (CDC) , (2020). Measles

[3] Majam A, Mulhee N and AL Kamarany Amood A, (2023). Measles Virus Infection Among Vaccinated and Unvaccinated Children in Hodeidah, Yemen , Workshop about Quality , Safety and Efficacy of Vaccines , 19 – 20 March , Page 15 .

[4] Griffin DE. Measles Vaccine (2018). *Viral Immunol*. 31(2):86-95.

[5] Guerra FM, Bolotin S, Lim G, Heffernan J, Deeks SL, Li Y, Crowcroft NS (2017). The basic reproduction number (R0) of measles: a systematic review. *Lancet Infect Dis*. 17(12):e420-e428.

[6] Plans Rubio P (2012). Is the basic reproductive number (R(0)) for measles viruses observed in recent outbreaks lower than in the pre-vaccination era? Euro Surveill. 2;17(31):22

[7] Bakare E.A., Adekunle Y.A. , Kadiri K.O., (2012). Modelling and simulation of the dynamics of the transmission of measles. International Journal of Computer Trends and Technology, vol. 3, pp. 174-178.

[8] Momoh A.A., Ibrahim M.O., Uwanta I.J., Manga S.B., (2013), Mathematical model for control of measles epidemiology. International Journal of Pure and Applied Mathematics, 87(5), 707-718.

[9] Tessa O.M., (2006), Mathematical model for control of measles by vaccination. In Proceedings of Mali Symposium on Applied Sciences, vol. 2006 : 31-36.

[10] Kilbas A.A., Srivastava H.M., and Trujillo J.J. (2006), "Theory and Applications of Fractional Differential Equations", Elsevier B.V., Amsterdam, The Netherlands.

[11] Caputo M., and Fabrizio M. (2015), "New definition of fractional derivative without singular kernel," *Prog. Fract. Differ. Appl.*, 1 (2): 73-85.

[12] Losada J., and Nieto J. (2015), "Properties of a new fractional derivative without singular kernel," *Prog. Fract. Differ. Appl.*, 1 (2): 87—92.

[13] Atangana A., and Baleanu D. (2016), "New fractional derivative with non-local and non-singular kernel," *Therm. Sci.*, 20 (2): 757-763.

[14] Atangana A., and Araz S.I. (2021), New concept in calculus: Piecewise differential and integral operators. Chaos Solitons Fractals 2021; 145: 110638.