



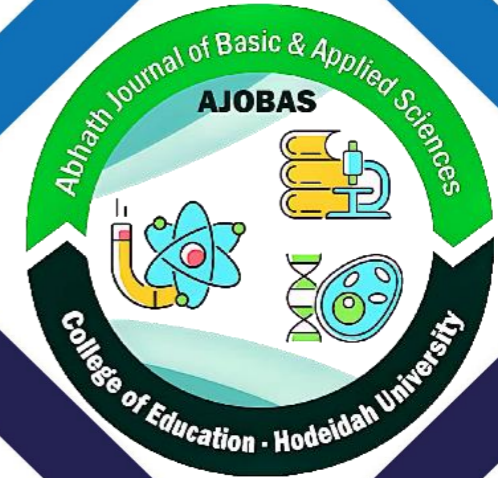
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# **ABHATH**

## **Journal of Basic and Applied Sciences**

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## **Introduction of the Issue**

We are pleased and delighted to present the researchers with this issue of the 'Abhath' Journal of Basic and Applied Sciences, which is the first issue of the first volume, the issuance of which emanates as an affirmation of moving forward towards issuing specialized quality journals.

The Faculty of Education at Hodeidah University aims, by issuing this journal, to publish specialized researches in basic and applied sciences, from inside and outside Yemen, in the English language.

On this occasion, the journal invites male and female researchers to submit their researches for publication in the next issues of the journal.

In conclusion, the editorial board of the journal extends its thanks and gratitude to Prof. Mohammed Al-Ahdal – Rector of the university – the general supervisor of the journal, for his support and encouragement for the establishment of this journal. Furthermore, thanks are extended to Prof. Mohammed Bulghaith – University Vice-Rector for Higher Studies and Scientific Research – vice-supervisor of the journal, for his cooperation in facilitating the procedures for the issuance of this issue. Nevertheless, thanks are for all researchers whose scientific articles were published in this issue, and for the editorial board of the journal, which worked tirelessly to produce this issue in this honorable way.

### **Journal Chief Editor**

Prof. Yusuf Al-Ojaily



## **Modified ELzaki Transform and its Applications**

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### **ABSTRACT.**

Modified Laplace transform and modified Sumudu transform were suggested by Saif et al. [1] and Ugur Duran [2], respectively, they discussed some of characteristics and theorems. Motivated by this kind of work, the Modified ELzaki transform that is an extension of the ELzaki integral transform for solving differential equations, is introduced in his study. The modified integral transform is successfully deduced from the ELzaki transform and examine many properties and relations as well as used to ordinary and partial differential equations to demonstrate its simplicity, efficiency, and high accuracy.

### **1. INTRODUCTION.**

Under acceptable initial conditions, integral transforms are one of the most significant techniques for solving ordinary and partial differential equations. P. S. Laplace's work in the 1780s and Joseph Fourier's work in 1822 are the originators of the integral transforms. They are two of the most widely used transformations in the literature. For a piecewise continuous function  $f(t)$  of exponential order, the Laplace integral transform is defined as,



$$\begin{aligned}\mathcal{L}\{f(t)\} = F(s) &= \int_0^{\infty} e^{-st} f(t) dt & s > 0 & \quad (1) \\ &= \lim_{\tau \rightarrow \infty} \int_0^{\tau} e^{-st} f(t) dt\end{aligned}$$

provided that the limit of integral exists. while the Fourier integral transform which is similar with Laplace integral transform is mathematically expressed as:

$$\mathcal{F}\{f(t)\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-ist} dt \quad (2)$$

There are many integral transforms in the Laplace and Fourier transforms class have been introduced in the recent two decades, including Laplace-Carson transform [3], Sumudu transform [4], Natural transform [5], Elzaki transform [6], Aboodh transform [7], The novel integral transform [8], Yang transform [9,11], and Shehu transform [12]. These transforms have a wide range of applications in physics, electrical engineering, control engineering, optics, mathematics, and signal processing, to name a few. In recent years, most of the integral transforms have been modified, improved and generalization such as Modified Laplace transform [1], Modified Sumudu Transform [2], z-transform as the discrete-time equivalent of the Laplace transform [13], The Mellin integral transform as multiplicative version of the two-sided Laplace transform, Laplace-Carson transform [3], degenerate Elzaki transform, n-dimensional Laplace transforms.

The Elzaki Transform which was proposed by Elzaki [1] as a class of Laplace transforms is useful for solving a variety of ordinary and partial differential equations, as well as integral equations. It's also a useful tool for building analytic approximate solutions to scientific problems.

Motivated by the above-mentioned researches, in this paper we proposed a Modified ELzaki transform for solving both ordinary and partial

differential equations. Where for  $a = e$ , the Modified ELzaki transform converges to the ELzaki transform. Both ordinary and partial differential equations can be solved using the Modified integral transform. The Modified ELzaki transform is indicated by the operator  $E_a[f(t)]$  throughout this research.

in [14], Apply Elzaki transform for partial derivatives and its applicability. It is also has been used in [15] to solve a variety of mathematical problems that arise in the fields of physics and finance. In [16], this transform has been effectively applied to locate analytical solutions for linear Volterra type integral problems. The quadratic Riccati differential equation was solved by the Elzaki transform method [17]. It has been shown also in [18] how the ELzaki transform may be used to solve an ordinary differential equation with variable coefficients.

## 2. Modified Elzaki Transform

In [1,2], the modified Laplace transforms and modified Sumudu transforms of a function  $f(t)$  which is piece-wise continuous and of exponential order is considered as follows:

$$\mathcal{L}_a[f(t)] = F(s) = \int_0^{\infty} a^{-st} f(t) dt; \quad \mathcal{R}(s) > 0, a > 0, \\ a \neq 1 \text{ and} \quad (3)$$

$$S_a[f(t)] = \int_0^{\infty} a^{-\frac{1}{u}t} f(t) dt; \quad u > 0, a > 0, \\ a \neq 1 \quad (4)$$

Respectively, Note that for  $a = e$  is set, the modified Laplace transform in (1) and the modified Sumudu transform in (2) are both reduced to the standard Laplace and Sumudu transform. Motivated by the above, we define modified ELzaki transform as following:

$$E_a[f(t)] = v \int_0^\infty a^{-\frac{1}{v}t} f(t) dt; \quad v > 0, a > 0, a \neq 1 \tag{5}$$

We note that

$$E_e[f(t)] = E[f(t)]$$

The inverse Modified Elzaki transform is given by

$$E_a^{-1}[f(t)] = \frac{1}{2\pi i} \int_0^\infty a^{\frac{1}{v}t} F_a(v) dt; \quad v > 0, a > 0, a \neq 1 \tag{6}$$

### 3. Modified Elzaki Transform of Some Functions

3.1. Let  $f(t) = 1$ , By Definition 1, we observe that

$$\begin{aligned} E_a[1] &= v \int_0^\infty a^{-\frac{1}{v}t} dt \\ &= v \left( -\frac{v}{\ln a} \right) a^{-\frac{1}{v}t} \Big|_0^\infty \\ &= \frac{v^2}{\ln a} \end{aligned}$$

3.2. Let  $f(t) = t$ , By Definition 1, we observe that

$$\begin{aligned} E_a[t] &= v \int_0^\infty a^{-\frac{1}{v}t} t dt \\ &= v \left( \left( -\frac{v}{\ln a} t a^{-\frac{1}{v}t} \right) \Big|_0^\infty + \frac{v}{\ln a} \int_0^\infty a^{-\frac{1}{v}t} dt \right) \\ &= v \left( 0 + \frac{v}{\ln a} \left( \frac{v}{\ln a} \right) \right) \\ &= \frac{v^3}{(\ln a)^2} \end{aligned}$$

3.3. Let  $f(t) = t^n, n = 0,1,2, \dots$ , By Definition 1, we observe that

$$E_a[t] = v \int_0^\infty a^{-\frac{1}{v}t} t^n dt$$

Let  $I_n = v \int_0^\infty a^{-\frac{1}{v}t} t^n dt$ , then  $I_0 = \frac{v^2}{\ln a}$ ,  $I_1 = \frac{v^3}{(\ln a)^2}$  and

$$\begin{aligned}
 I_n &= \frac{v}{\ln a} n I_{n-1} \\
 &= \frac{v}{\ln a} \frac{v}{\ln a} n (n-1) I_{n-2} \\
 &= \frac{v}{\ln a} \frac{v}{\ln a} \frac{v}{\ln a} n (n-1)(n-2) I_{n-3} \\
 &= \frac{v}{\ln a} \frac{v}{\ln a} \frac{v}{\ln a} n (n-1)(n-2) \dots \dots \dots 2 \times 1 \times I_0 \\
 &= \frac{v}{\ln a} n \frac{v}{\ln a} (n-1) \frac{v}{\ln a} (n-2) \dots \dots \dots \frac{v}{\ln a} 2 \times \frac{v}{\ln a} 1 \\
 &\quad \times \frac{v^2}{\ln a} \\
 &= \frac{v^n}{(\ln a)^n} n! \frac{v^2}{\ln a} = \frac{v^{n+2}}{(\ln a)^{n+1}} n! , \quad \frac{1}{v} \\
 &\quad > 0 \text{ and } \frac{1}{\ln a} > 0
 \end{aligned}$$

3.4. Let  $f(t) = t^p$ ,  $p \in R, p \neq -1, -2, -3, \dots$ , By Definition 1, we observe that

$$\begin{aligned}
 E_a[t^p] &= v \int_0^\infty a^{-\frac{1}{v}t} t^p dt \\
 &= v \int_0^\infty e^{\ln a \frac{1}{v}t} t^p dt \\
 &= v \int_0^\infty e^{-\frac{1}{v}t \ln a} t^p dt
 \end{aligned}$$

Let  $z = \frac{1}{v}t \ln a$ , then  $t = \frac{v}{\ln a}z$ ,  $dt = \frac{v}{\ln a} dz$  and

$$\begin{aligned}
 E_a[tt^p] &= v \int_0^\infty e^{-z} \left(\frac{v}{\ln a}z\right)^p \frac{v}{\ln a} dz \\
 &= \frac{v^{p+2}}{(\ln a)^{p+1}} \int_0^\infty e^{-z} z^{p+1-1} dz \\
 &= \frac{v^{p+2}}{(\ln a)^{p+1}} \Gamma(p+1)
 \end{aligned}$$

Where,  $\Gamma$  is Gama Function.

3.5. Let  $f(t) = a^{pt}$ , By Definition 1, we observe that

$$\begin{aligned} E_a[a^{pt}] &= v \int_0^\infty a^{pt} a^{-\frac{1}{v}t} dt \\ &= v \int_0^\infty a^{-(\frac{1}{v}-p)t} dt \\ &= v \left( -\frac{1}{\left(\frac{1}{v}-p\right) \ln a} a^{-(\frac{1}{v}-p)t} \right) \Bigg|_0^{-\infty} \\ &= \frac{v^2}{(1-vp) \ln a}, \quad \frac{1}{v} > p \end{aligned}$$

3.6. Let  $f(t) = e^{pt}$ , By Definition 1, we observe that

$$\begin{aligned} E_a[a^{pt}] &= v \int_0^\infty e^{pt} a^{-\frac{1}{v}t} dt \\ &= v \int_0^\infty e^{pt} e^{\ln a^{-\frac{1}{v}t}} dt \\ &= v \int_0^\infty e^{-(\frac{\ln a}{v}-p)t} dt \\ &= v \left( -\frac{1}{\left(\frac{\ln a}{v}-p\right)} e^{-(\frac{\ln a}{v}-p)t} \right) \Bigg|_0^{-\infty} \\ &= \frac{v^2}{(\ln a - vp)}, \quad \frac{\ln a}{v} > p \end{aligned}$$

3.7. Let  $f(t) = \sin bt$ , By Definition 1, we observe that

$$\begin{aligned} E_a[a^{pt}] &= v \int_0^\infty a^{-\frac{1}{v}t} \sin bt dt \\ &= v \int_0^\infty a^{-\frac{1}{v}t} \left( \frac{e^{ibt} - e^{-ibt}}{2i} \right) dt \\ &= \frac{1}{2i} \left( v \int_0^\infty e^{ibt} a^{-\frac{1}{v}t} dt - v \int_0^\infty e^{-ibt} a^{-\frac{1}{v}t} dt \right) \\ &= \frac{1}{2i} \left( \frac{v^2}{(\ln a - ivb)} - \frac{v^2}{(\ln a + ivb)} \right) \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2i} \frac{2ibv^3}{((\ln a)^2 + v^2b^2)}, \\
 &= \frac{bv^3}{((\ln a)^2 + v^2b^2)}
 \end{aligned}$$

3.8. Let  $f(t) = \cos bt$ , By Definition 1, we observe that

$$\begin{aligned}
 E_a[a^{pt}] &= v \int_0^\infty a^{-\frac{1}{v}t} \cos bt \, dt \\
 &= v \int_0^\infty a^{-\frac{1}{v}t} \left( \frac{e^{ibt} + e^{-ibt}}{2} \right) dt \\
 &= \frac{1}{2} \left( v \int_0^\infty e^{ibt} a^{-\frac{1}{v}t} dt \right. \\
 &\quad \left. + v \int_0^\infty e^{-ibt} a^{-\frac{1}{v}t} dt \right) \\
 &= \frac{1}{2i} \left( \frac{v^2}{(\ln a - ivb)} + \frac{v^2}{(\ln a + ivb)} \right) \\
 &= \frac{1}{2} \frac{2 \ln a \, v^2}{((\ln a)^2 + v^2b^2)}, \\
 &= \frac{\ln a \, v^2}{((\ln a)^2 + v^2b^2)}
 \end{aligned}$$

3.9. Let  $f(t) = \sinh bt$ , By Definition 1, we observe that

$$\begin{aligned}
 E_a[a^{pt}] &= v \int_0^\infty a^{-\frac{1}{v}t} \sinh bt \, dt \\
 &= v \int_0^\infty a^{-\frac{1}{v}t} \left( \frac{e^{bt} - e^{-bt}}{2} \right) dt \\
 &= \frac{1}{2} \left( v \int_0^\infty e^{bt} a^{-\frac{1}{v}t} dt \right. \\
 &\quad \left. - v \int_0^\infty e^{-bt} a^{-\frac{1}{v}t} dt \right) \\
 &= \frac{1}{2i} \left( \frac{v^2}{(\ln a - vb)} - \frac{v^2}{(\ln a + vb)} \right) \\
 &= \frac{1}{2} \frac{2bv^3}{((\ln a)^2 - v^2b^2)},
 \end{aligned}$$

$$= \frac{b v^3}{((\ln a)^2 - v^2 b^2)}$$

3.10. Let  $f(t) = \cosh bt$ , By Definition 1, we observe that

$$\begin{aligned} E_a[a^{vt}] &= v \int_0^\infty a^{-\frac{1}{v}t} \cosh bt \, dt \\ &= v \int_0^\infty a^{-\frac{1}{v}t} \left( \frac{e^{bt} + e^{-bt}}{2} \right) dt \\ &= \frac{1}{2} \left( v \int_0^\infty e^{bt} a^{-\frac{1}{v}t} dt + v \int_0^\infty e^{-bt} a^{-\frac{1}{v}t} dt \right) \\ &= \frac{1}{2i} \left( \frac{v^2}{(\ln a - vb)} + \frac{v^2}{(\ln a + vb)} \right) \\ &= \frac{1}{2} \frac{2 \ln a v^2}{((\ln a)^2 - v^2 b^2)}, \\ &= \frac{\ln a v^2}{((\ln a)^2 - v^2 b^2)} \end{aligned}$$

3.11. Let  $f(t) = u(t - b)$ , where  $u(t - b)$  is unit step function defined by

$$u(t - b) = \begin{cases} 0, & 0 \leq t < b \\ 1, & b < t \end{cases}$$

By Definition 1, we observe that

$$\begin{aligned} E_a[u(t - b)] &= v \int_b^\infty a^{-\frac{1}{v}t} 1 \, dt \\ &= -\frac{v^2}{\ln a} \left( 0 - a^{-\frac{1}{v}b} \right) = \frac{v^2}{\ln a} a^{-\frac{1}{v}b} \end{aligned}$$

#### 4. The properties and Theorems of the Modified Elzaki Transform.

In the following theorems, we state and explore the general characteristics of the Sumudu transform:

**Linear property of modified Laplace transform:** The modified Elzaki transform is a linear transform, namely,

**Theorem 1.** Let,  $k_1, k_2$  be constants and  $E_a[f(t)] = F(v), E_a[g(t)] = G(v)$ , be the modified Elzaki transform of  $f(t), g(t)$ , respectively, then

$$\begin{aligned} E_a[k_1f(t) + k_2g(t)] &= v \int_0^\infty a^{-\frac{1}{v}t} (k_1f(t) + k_2g(t)) dt \\ &= k_1v \int_0^\infty a^{-\frac{1}{v}t} f(t) dt + k_2v \int_0^\infty a^{-\frac{1}{v}t} g(t) dt \end{aligned}$$

**Property 1.** (Scalar property) let  $E_a[f(t)] = F(v)$ , then  $E_a[f(bt)] = \frac{1}{b^2} F(bv)$

**Proof.** By definition

$$\begin{aligned} E_a[f(bt)] &= v \int_0^\infty a^{-\frac{1}{v}t} f(bt) dt \\ \text{Putting } z = bt, \text{ we get } t &= \frac{z}{b} \text{ and } dt = \frac{dz}{b} \text{ then} \\ E_a[f(bt)] &= v \int_0^\infty a^{-\frac{1}{v}(\frac{z}{b})} f(z) \frac{dz}{b} \\ &= \frac{1}{b^2} vb \int_0^\infty a^{-\frac{1}{vb}z} f(z) dz \\ &= \frac{1}{b^2} F(vb), \frac{1}{vb} > 0 \end{aligned}$$

**Theorem 2.** (First shifting Theorem): let  $E_a[f(t)] = F(v)$ , then

$$E_a[a^{bt}f(t)] = (1 - bv)F\left(\frac{v}{1 - bv}\right)$$

**Proof.** By definition

$$\begin{aligned} E_a[a^{bt}f(t)] &= v \int_0^\infty a^{bt} a^{-\frac{1}{v}t} f(t) dt \\ &= v \int_0^\infty a^{-\left(\frac{1}{v}-b\right)t} f(t) dt \\ &= (1 - bv) \frac{v}{1 - bv} \int_0^\infty a^{-\left(\frac{1}{1-bv}\right)t} f(t) dt \\ &= (1 - bv)F\left(\frac{v}{1 - bv}\right) \end{aligned}$$

**Theorem 3.** (Second shifting Theorem) let  $E_a[f(t)] = M(v)$ , then

$$E_a[f(t - b)u(t - b)] = a^{-\frac{1}{v}b} F(v)$$



Proof. By definition

$$E_a[f(t - b)u(t - b)] = v \int_b^\infty a^{-\frac{1}{v}t} f(t - b) dt$$

Putting  $z = t - b$ , then  $t = b \rightarrow z = 0$ ,  $t = \infty \rightarrow z = \infty$  and  $t = z + b$  and  $dt = dz$ , so

$$\begin{aligned} E_a[f(t - b)u(t - b)] &= a^{-\frac{1}{v}b} \left( v \int_0^\infty a^{-\frac{1}{v}z} f(z) dz \right) \\ &= a^{-\frac{1}{v}b} F(v) \end{aligned}$$

**Theorem 4.** (The Modified Elzaki transform of derivative) let  $E_a[f(t)] = F(v)$ , then  $E_a[f'(t)] = \frac{\ln a}{v} F(v) - v f(0)$

Proof. By definition

$$\begin{aligned} E_a[f'(t)] &= v \int_0^\infty a^{-\frac{1}{v}t} f'(t) dt \quad \text{use integration by parts} \\ &= v \left[ a^{-\frac{1}{v}t} f(t) \Big|_0^\infty + \frac{\ln a}{v} \int_0^\infty a^{-\frac{1}{v}t} f(t) dt \right] \\ &= v \left[ -f(0) + \frac{\ln a}{v^2} F(v) \right] \\ &= \frac{\ln a}{v} F(v) - v f(0) \end{aligned}$$

Similarly,  $E_a[f''(t)] = \left(\frac{\ln a}{v}\right)^2 F(v) - \ln a f(0) - v f'(0)$  and then  $E_a[f^{(n)}(t)]$

$$\begin{aligned} &= \left(\frac{\ln a}{v}\right)^n F(v) \\ &\quad - \frac{1}{v^n} \sum_{i=0}^n (\ln a)^{n-i-1} v^{i+2} f^{(i)}(0) \end{aligned}$$

**Theorem 4.** (convolution ) Let  $E_a[f(t)] = F(v)$ , then

$$E_a[f * g] = \frac{1}{v} E_a[f(t)] E_a[g(t)]$$

Proof. By definition

$$E_a[f * g] = v \int_0^\infty a^{-\frac{1}{v}t} f * g(t) dt$$

$$= v \int_0^\infty a^{-\frac{1}{v}t} \int_0^\infty f(\tau) g(t - \tau) d\tau dt$$

By interchange the order of integration, we get

$$E_a[f * g] = v \int_0^\infty f(\tau) \int_\tau^\infty a^{-\frac{1}{v}t} g(t - \tau) dt d\tau$$

Putting  $z = t - \tau$ , then  $t = \tau \rightarrow z = 0$ ,  $t = \infty \rightarrow z = \infty$  and  $t = z + \tau$  and  $dt = dz$ , so

$$\begin{aligned} &= v \int_0^\infty f(\tau) a^{-\frac{1}{v}\tau} \int_0^\infty a^{-\frac{1}{v}z} g(z) dz d\tau \\ &= \frac{1}{v} v \int_0^\infty f(\tau) a^{-\frac{1}{v}\tau} E_a[g] d\tau \\ &= \frac{1}{v} E_a[f] E_a[g] \end{aligned}$$

Table 1 is a collection of these elementary functions' Modified Elzaki transforms.

Table 1: Modified Elzaki Transforms for elementary functions

Function $f(t)$	$E_a[f(t)]$
1	$\frac{v^2}{\ln a}$
$t$	$\frac{v^3}{(\ln a)^2}$
$t^n$	$\frac{v^{n+2} n!}{(\ln a)^{n+1}} \frac{1}{v} > 0, \ln a > 0, n = 0, 1, 2, \dots$
$t^b$	$\frac{v^{b+2} \Gamma(b+1)}{(\ln a)^{b+1}} \frac{1}{v} > 0, \ln a > 0, b \in \mathbb{R},$ $b \neq -2, -3, \dots$
$a^{bt}$	$\frac{v^2}{\ln a - bv}, \frac{\ln a}{v} - b > 0$

$\sin bt$	$\frac{b v^3}{(\ln a)^2 - (bv)^2}, \quad \frac{\ln a}{v} > b$
$\cos bt$	$\frac{\ln a v^2}{(\ln a)^2 + (bv)^2}$
$\sinh bt$	$\frac{b v^3}{(\ln a)^2 - (bv)^2}, \quad \frac{\ln a}{v} > b$
$\cosh bt$	$\frac{\ln a v^2}{(\ln a)^2 - (bv)^2}, \quad \frac{\ln a}{v} > b$

### 5. Inverse of Modified ELzaki Transform

In view of (5) , (6) and definition of the above transforms which are listed in the

Table 1, we have:

$$E_a[\cos bt] = \frac{\ln a v^2}{(\ln a)^2 + (bv)^2}$$

$$E_a^{-1} \left[ \frac{v^2}{\ln a} \right] = 1$$

$$E_a^{-1} \left[ \frac{v^3}{(\ln a)^2} \right] = t$$

$$E_a^{-1} \left[ \frac{v^{p+2}}{(\ln a)^{p+1}} \Gamma(p + 1) \right] = t^n$$

$$E_a^{-1} \left[ \frac{v^{p+2}}{(\ln a)^{p+1}} n! \right] = t^n$$

$$E_a^{-1} \left[ v \frac{1}{\left(\frac{1}{v} - p\right) \ln a} \right] = a^{pt}$$

$$E_a^{-1} \left[ v \frac{1}{\left(\ln a \frac{1}{v} - p\right)} \right] = e^{pt}$$

$$E_a^{-1} \left[ \frac{bv^3}{((\ln a)^2 + v^2 b^2)} \right] = \sin bt$$

$$E_a^{-1} \left[ \frac{\ln a v^2}{((\ln a)^2 + v^2 b^2)} \right] = \cos bt$$

$$E_a^{-1} \left[ \frac{bv^3}{((\ln a)^2 - v^2 b^2)} \right] = \sinh bt$$

$$E_a^{-1} \left[ \frac{\ln a v^2}{((\ln a)^2 - v^2 b^2)} \right] = \cosh bt$$

6. Applications

6.1. Application of Modified ELzaki Transform to Simple Harmonic Motion.

Let us consider a particle of mass ( $m$ ) traveling in a straight line along the path AB under the action of a restoring force equal to  $K$ , with a position of equilibrium at  $O$  [19].

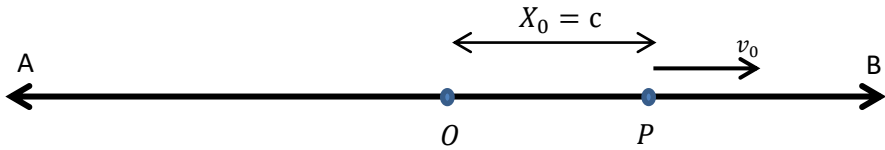


Figure 1 (a) at  $t = t_0$

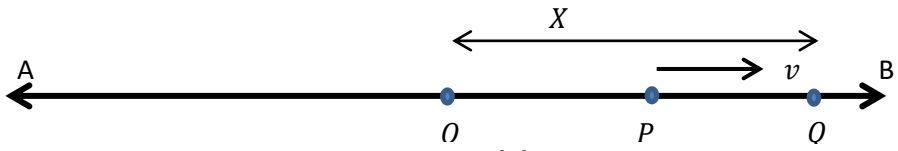


Figure 1 (b) at  $t = t$

Let :

$X$  – is displacement from a position of equilibrium.

$v$  – is particle's speed from  $P$  where  $v = \frac{dX}{dt}$

$v_0$  – is particle's initial velocity,  $v(0)$

$K$  – is restoring force under which particle moving (positive constant)

$a$  – particle's acceleration from  $P$ , and equal to is  $\frac{d^2X}{dt^2}$ .

$\vec{F}$  – The restoring force is given by  $\vec{F} = -k x$

This force is equal to the particle's mass times its acceleration in the positive  $X$ -direction, according to Newton's equation of motion. i.e.  $\vec{F} = m a$ , Let the particle starts from rest at distance  $c$  from the equilibrium. The equation governing particle's motion is given by

$$X'' + \omega^2 X = 0 \quad (7)$$

With  $X(0) = c$  and  $X'(0) = 0$

Where  $\omega^2 = \frac{K}{m}$ ,

Applying Modified Elzaki transform on both sides of Eq. (7) we have

$$\left(\frac{\ln a}{v}\right)^2 E_a[X] - \ln a X(0) - v X'(0) + \omega^2 E_a[X] = 0$$

Or

$$\left(\left(\frac{\ln a}{v}\right)^2 + \omega^2\right) E_a[X] = c \ln a$$

which on applying the initial condition and simplification give,

$$E_a[X] = \frac{c v^2}{(\ln a)^2 + (v\omega)^2} \ln a$$

The inverse Modified Elzaki transform gives,

$$X = c \cos \omega t \quad \text{or} \quad \cos \omega t = \frac{X}{c}$$

So, the velocity is

$$v = X' = -c \omega \sin \omega t = -c\omega \sqrt{1 - \cos^2 \omega t} \quad \text{or} \\ v = -\omega \sqrt{c^2 - X^2}$$

It is clear that the particle attains its maximum velocity at  $0$  i.e.

At  $x = 0$ , so,

$$v_{max} = -\omega c$$

## 6.2 Application of Modified ELzaki Transform to Forced Undamped vibration.

In contrast to natural vibration, undamped forced vibration is the consequence of ongoing external stimuli. It is regarded as one of the most significant kind of vibration. Its basic idea may be used to explain how any kind of machine or building moves [19].

Let

$m$ - is mass attached to a spring.

$K$ - is a constant of proportionality called the spring constant.

$\vec{F}$  – The restoring force is given by  $\vec{F} = -k x$ , force also equals mass times acceleration, according to Newton's equation of motion. i.e.

$\vec{F} = m a$ , The equation of motion of such system will be:

$$X'' + \omega^2 X = F \sin nt \tag{8}$$

With  $X(0) = 0$  and  $X'(0) = 0$ , Applying Modified Elzaki transform on both sides of Eq. (8) we have

$$E_a[X'' + \omega^2 X] = E_a[F \sin nt]$$

Or

$$\left(\frac{\ln a}{v}\right)^2 E_a[X] - \ln a X(0) - v X'(0) + \omega^2 E_a[X] = F \frac{n v^3}{(\ln a)^2 + (vn)^2}$$

Thus, after simplifying,

$$\left(\left(\frac{\ln a}{v}\right)^2 + \omega^2\right) E_a[X] = F \frac{n v^3}{(\ln a)^2 + (vn)^2}$$

Rearranging the equation, yields the following:

$$E_a[X] = F \frac{v^2}{(\ln a)^2 + (v\omega)^2} \frac{n v^3}{(\ln a)^2 + (vn)^2}$$

The inverse Modified Elzaki transform gives,

$$X = \frac{F}{\omega} E_a^{-1} \left[ \frac{1}{v} \frac{\omega v^3}{(\ln a)^2 + (v\omega)^2} \frac{n v^3}{(\ln a)^2 + (vn)^2} \right]$$

Thus,

$$X = \frac{F}{\omega} \sin \omega t * \sin nt$$

Which for  $n = \omega$  becomes ,

$$\begin{aligned} X &= \frac{F}{\omega} \sin \omega t * \sin \omega t \\ &= \frac{F}{\omega} \int_0^t \sin \omega \tau \sin(\omega t - \omega \tau) d\tau \\ &= \frac{F}{\omega} \int_0^t (\cos(2\omega \tau - \omega t) - \cos(\omega t)) d\tau \end{aligned}$$

$$= \frac{F}{2\omega} \left[ \frac{1}{\omega} \sin \omega t - t \cos \omega t \right]$$

**6.3 Application of Modified ELzaki Transform to Deflection of Beams.**

Consider the axially loaded strut subjected to equal and opposite compressive forces at its ends and the crippling load P, which satisfies the equation [20].

$$y'' + n^2y = A_1x$$

With  $y(0) = 0$  and  $y'(\frac{1}{2}) = 0$ . Where  $A_1 = -\frac{w}{2p}n^2$ , Apply modified Elzaki transform to both sides, and used the initial condition  $y(0) = 0$ , we have

$$E_a[y'' + n^2y] = E_a[A_1x]$$

Or

$$\begin{aligned} \left(\frac{\ln a}{v}\right)^2 E_a[y] - \ln a f(0) - v f'(0) + n^2 E_a[y] \\ = A_1 \frac{v^3}{(\ln a)^2} \end{aligned}$$

Rearranging the equation, yields the following:

$$\begin{aligned} E_a[y] \\ = A_1 \frac{v^2}{(\ln a)^2 + (vn)^2} \frac{v^3}{(\ln a)^2} \\ + f'(0) \frac{v^3}{(\ln a)^2 + (vn)^2} \end{aligned}$$

The inverse Modified Elzaki transform gives,

$$\begin{aligned} y \\ = \frac{A_1}{n} E_a^{-1} \left[ \frac{1}{v} \frac{nv^3}{(\ln a)^2 + (vn)^2} \frac{v^3}{(\ln a)^2} \right] \\ + \frac{f'(0)}{n} E_a^{-1} \left[ \frac{nv^3}{(\ln a)^2 + (vn)^2} \right] \\ = \frac{A_1}{n} (x * \sin nx) + \frac{f'(0)}{n} \sin nx \\ = \frac{A_1}{n} \int_0^x \tau \sin(nx - n\tau) d\tau + \frac{f'(0)}{n} \sin nx \end{aligned}$$

$$\begin{aligned}
 &= \frac{A_1}{n} \int_0^x \tau (\sin nx \cos n\tau \\
 &\quad - \cos nx \sin n\tau) d\tau \\
 &\quad + \frac{f'(0)}{n} \sin nx \\
 &= \frac{A_1}{n} \sin nx \int_0^x \tau \cos n\tau d\tau \\
 &\quad - \frac{A_1}{n} \cos nx \int_0^x \tau \sin n\tau d\tau \\
 &\quad + \frac{f'(0)}{n} \sin nx \\
 &= \frac{A_1}{n} \sin nx \left[ \frac{x}{n} \sin nx + \frac{1}{n^2} \cos nx - 1/n^2 \right] \\
 &\quad - \frac{A_1}{n} \cos nx \left[ -\frac{x}{n} \cos nx \right. \\
 &\quad \left. + \frac{1}{n^2} \sin nx \right] + \frac{f'(0)}{n} \sin nx \\
 &= \frac{A_1}{n} \left[ \frac{x}{n} - \frac{1}{n^2} \sin nx \right] + \frac{f'(0)}{n} \sin nx
 \end{aligned}$$

Thus ,

$$\begin{aligned}
 y &= \frac{A_1}{n} \frac{x}{n} + \left( \frac{f'(0)}{n} - \frac{A_1}{n} \frac{1}{n^2} \right) \sin nx \\
 &= \frac{A_1}{n} \frac{x}{n} + A_2 \sin nx
 \end{aligned}$$

Put  $A_2 = \left( \frac{f'(0)}{n} - \frac{A_1}{n} \frac{1}{n^2} \right)$

Now, since

$$y'(x) = \frac{A_1}{n^2} + A_2 n \cos nx$$

Applying the second initial condition  $y\left(\frac{1}{2}\right) = 0$ ,

we get

$$\begin{aligned}
 0 &= \frac{A_1}{n^2} + A_2 n \cos \frac{nL}{2} \rightarrow A_2 \\
 &= -\frac{A_1}{n^2} \frac{1}{n \cos \frac{nL}{2}}
 \end{aligned}$$



$$\rightarrow A_2 = -\frac{1}{n^2} \left( -\frac{w}{2p} n^2 \right) \frac{1}{n \cos \frac{nL}{2}} = \frac{w}{2p} \frac{1}{n \cos \frac{nL}{2}}$$

Thus, the final solution is

$$y = \frac{w}{2p} \frac{\sin nx}{n \cos \frac{nL}{2}} - \frac{w}{2p} x$$

## 7. Conclusions.

Elzaki transform is an effective tool for creating analytic approximate solution of scientific problems. Elzaki [6] first proposed it as a variation of the traditional Sumudu and Laplace transform. For the quickest computation time compared to other methods, this method is highly recommended for solving ordinary and partial linear and non-linear differential equations. As a follow-up, the purpose of this paper is to propose a modified version of the Elzaki transform that is more general and an efficient for solving differential equations. Modified ELzaki Transform of some elementary functions are introduced. We presented its existence and inverse transform as well as discussion of the general properties of the Transform. We applied the Modified ELzaki Transform to some class of ordinary differential equations which appearing in physical and engineering applications. The result confirms that the Modified ELzaki Transform technique is preferable to other transforms

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