ISSN:0000-0000

Abhath JOURNAL of Basic and Applied Sciences

Abhath Journal of Basic & Applied Sciences
AJOBAS
College of Education - Hodeidah University

A semi-annual peer reviewed scientific journal published by the College of Education Hodeidah University

Volume 1, Number 1, June 2022

Abhath JOURNAL of Basic and Applied Sciences Volume 1, Number 1, June 2022

جامعة الحديدة
HODIEDAH UNIVERSITY
كلية التربية بالحديدة
F.E.H

ABHATH

Journal of Basic and Applied Sciences

A Semi-Annual Peer - Reviewed Scientific Journal

Issued by the Faculty of Education in Hodeidah – Hodeidah University

Website:
https://ajobas.com

First Volume – First Issue (June 2022)

ABHATH

Journal of Basic and Applied Sciences

An arbitrated scientific journal specialized in basic and applied sciences that publishes on its pages the products of various research works, characterized by originality and add to knowledge what researchers in all branches of basic and applied sciences can benefit from.

The journal receives scientific researches from all countries of the world in English. The researches are then arbitrated by specialized arbitrators.

The journal is published semi-annually with two issues per year.

Deposit No. at the 'House of Books' in Sana'a: (159/1443-2022)

Correspondences to be addressed to the Editorial Secretary name via the journal's E-mail or the mailing address below:

Abhath Journal for Basic and Applied Sciences – Faculty of Education – Hodeidah University

Hodeidah – Yemen Republic

P. O. Box (3114)

E-mail: abhath-journal@hoduniv.net.ye

Exchanges and Gifts: Requests to be addressed to the Editorial Secretary name

<u>Publishing Rules in the Journal</u>

- The research should be new and not published or before.
- The research should be written in English language.
- The research should represent a scientific addition in the field of Basic and Applied Sciences.
- Quality in idea, style, method, and scientific documentation, and without scientific and linguistic errors.
- The researcher must submit his/her CV.
- Sending the research to the journal is considered a commitment by the researcher not to publish the research in another journal.
- The researcher submits an electronic copy of the research in the **(Word)** format, sent via e-mail to the journal at**: <u>abhath-journal@hoduniv.net.ye</u>**; on it shall be written: **the title of the research, the name of the researcher (or researchers), along with the academic rank, current position, address, telephone and e-mail.**
- The researcher should follow the respected mechanisms and methods of scientific research.
- The research should be written in Times New Roman font, paper size: (17 width x 25 height), leaving a margin of 2 cm on all sides, Except for the The left side, 2.5 cm and 1.25 between lines, and subheadings must be bold. Font size 12 in the text, and 11 in the footnote.
- The drawings, tables, figures, pictures and footnotes (if any) should be well prepared.
- The researcher pays arbitration and publication fees for the research in the amount of (20,000 Yemeni riyals) for Yemeni researchers from inside Yemen, and (150 US dollars) for researchers from outside Yemen.
- The researcher submits an abstract with its translation in Arabic within (250) words, appended with keywords of no more than five words.
- The researcher submits a commitment not to publish the research or submit it for publication in another journal.
- The researcher submits his/her CV.
- The researcher is responsible for the validity and accuracy of the results, data and conclusions contained in the research.
- All published researches express the opinions of the researchers and does not necessarily reflect the opinion of the editorial board.

Exchanges and gifts: Requests to be addressed to the Editorial Secretary name.

Contents of the Issue

Introduction of the Issue

We are pleased and delighted to present the researchers with this issue of the 'Abhath' Journal of Basic and Applied Sciences, which is the first issue of the first volume, the issuance of which emanates as an affirmation of moving forward towards issuing specialized quality journals.

The Faculty of Education at Hodeidah University aims, by issuing this journal, to publish specialized researches in basic and applied sciences, from inside and outside Yemen, in the English language.

On this occasion, the journal invites male and female researchers to submit their researches for publication in the next issues of the journal.

In conclusion, the editorial board of the journal extends its thanks and gratitude to Prof. Mohammed Al-Ahdal – Rector of the university – the general supervisor of the journal, for his support and encouragement for the establishment of this journal. Furthermore, thanks are extended to Prof. Mohammed Bulghaith – University Vice-Rector for Higher Studies and Scientific Research – vice-supervisor of the journal, for his cooperation in facilitating the procedures for the issuance of this issue. Nevertheless, thanks are for all researchers whose scientific articles were published in this issue, and for the editorial board of the journal, which worked tirelessly to produce this issue in this honorable way.

Journal Chief Editor

Prof. Yusuf Al-Ojaily

A scientific refereed journal issued by Faculty of Education- Hodeidah University-Yemen

Abhath Journal of Basic and Applied Sciences
http://ajobas.com

Received: 20/5/2022. **Accepted:** 10/6/2022 **Vol. 1, No. 1, June 2022**

On Intuitionistic Fuzzy Separation Axioms

S. Saleh[1,2]

[1]Mathematics Department, Hodeidah University, Hodeidah, Yemen
[2] Computer Science Department, Cihan University-Erbil, Kurdistan, Iraq
E-mail: s_wosabi@hoduniv.net.ye

Abstract. In this paper we slightly alter Atanassov's definition of intuitionistic fuzzy sets (which are equivalent to interval valued fuzzy sets [11]) and we discuss some interesting new properties of intuitionistic fuzzy topology. The relation between IFTSs and the induced fuzzy bitopological spaces are studied. We also give the definitions of intuitionistic fuzzy regularity and separation axioms and give some its characterizations. We introduce new concept of good extension property in intuitionistic setting. Finally, we investigate some relations between separation axioms on IFTS and that of induced fuzzy topological spaces (FTSs for short) and vice versa.

Keywords. intuitionistic fuzzy sets, intuitionistic fuzzy point, intuitionistic fuzzy topology, intuitionistic product, intuitionistic fuzzy separation axioms, good extension.

1. Introduction and Preliminaries.

After the introduction of concept of fuzzy sets by Zadeh [18], many authors generalized the idea of fuzzy sets in different directions [1, 12, 16, 17]. Atanassov [1] introduced the concept of intuitionistic fuzzy sets as generalization of fuzzy sets. Later this concept was generalized to other aspects [2, 3, 4, 6]. Subsequently, Coker [6] introduced the concept of intuitionistic fuzzy topology and studied some of its properties [7-10].

Nevertheless, separation axioms in IFTSs are not studied. Only Coker and Bayhan [5] introduced several definitions of T_1 and T_2 and investigated the relation between them.

In this paper we introduce new concepts of intuitionistic fuzzy separation axioms, intuiti- onistic fuzzy regularity axioms in IFTS based on IFSs (in our sense). The relationship between our concepts and Coker-Bayhan concepts of T_1 and T_2 takes place in another article, because the definition of intuitionistic fuzzy point in both concepts is different.

In section.1 We will give a modification of IFS [1] investigating some new properties. **In section.2** We recall the definition of IFT. We introduce new concept, the intuitionistic product, we generate many fuzzy topologies from a given IFTS and vice versa, and we define some operators on it. **In section.3** We introduce the definitions of IF-separation axioms, IF-regularity axioms, we give some implications on it. The concept of the good extension property will be given and studied. **In section.4** We investigate some relations between separation axioms of IFTS and that of induced FTSs, some necessary counterexamples will be given. Finally, **in section.5** Some relations between separation axioms of fuzzy topology and that of induced intuitionistic fuzzy topological spaces (IFTSs) will be investigated, with some necessary counterexamples.

Atanassov [1] introduced the concept of intuitionistic fuzzy sets

Definition 1.1 [1] Let X be a nonempty fixed set. An intuitionistic fuzzy set A (IFS for short) is an object having the form:

$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$, where the functions μ_A : X $\to$ I and γ_A : X $\to$ I denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \gamma_A(x) \le 1 \ \forall \ x \in X$. Where I denotes unit interval [0,1].The family of all intuitionistic fuzzy sets on X, will be denoted by $I(X)$.

From the above definition it is clear that an intuitionistic fuzzy sets may be regarded as a pair $(A_1, A_2) \in I^X \times I^X$ such that $A_1 \subseteq A_2^C$, where A_2^C is the complement of A_2.

Accordingly, we can alter the Atanassov's definition of intuitionistic fuzzy set slightly to the following more convenient definition:

Definition 1.2

An intuitionistic fuzzy set $\underline{A}$ (IFS for short) is an ordered pair $\underline{A} = (A_1, A_2) \in I^X \times I^X$ such that $A_1 \subseteq A_2$. Where I^X is the family of all fuzzy sets on a given nonempty set X. The family of all intuitionistic fuzzy sets on X in this form, will be denoted by II^X,

i.e. $II^X = \{(A_1, A_2) : A_1, A_2 \in I^X \ and \ A_1 \subseteq A_2\}$. The IFS $\underline{X} = (X, X)$ is called universal IFS and the IFS $\underline{\phi} = (\phi, \phi)$ is called the empty IFS.

Any fuzzy set A on X is obviously an IFS in the form $\underline{A} = (A, A)$.

Definition1.3 Let $\underline{A} = (A_1, A_2)$, $\underline{B} = (B_1, B_2) \in II^X$. Then:

1) $\underline{A} = \underline{B} \Leftrightarrow A_i = B_i,\ i = 1, 2$,

2) $\underline{A} \subseteq \underline{B} \Leftrightarrow A_i \subseteq B_i,\ i = 1, 2$,

3) $\underline{A} \bigcup \underline{B} = (A_1 \bigcup B_1,\ A_2 \bigcup B_2)$,

4) $\underline{A} \bigcap \underline{B} = (A_1 \bigcap B_1,\ A_2 \bigcap B_2)$,

5) $\underline{A}^C = (A_2{}^C,\ A_1{}^C)$, where $\underline{A}^C$ is the complement of $\underline{A}$.

Remark.1.4 *By the canonical mapping from* $(I(X), \bigcup, \bigcap, C)$ *onto* $(II^X, \bigcup, \bigcap, C)$ *assigns for all* $A \in I(X)$ *the set* $(A_1, A_2^C) \in II^X$ *, we note that all results which are based on Atanassov's intuitionistic fuzzy sets still true in our setting.*

Definition. 1.5

Let X and Y be two nonempty sets and $f: X \to Y$ be a function,

i) If $\underline{A} = (A_1, A_2)$ is an IFS in X. Then the image of $\underline{A}$ under f, denoted by $f(\underline{A})$ is the IFS in Y, defined by, $f(\underline{A}) = (f(A_1),\ f(A_2))$, where

$$f(A_i)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} A_i(x) & if \quad f^{-1}(y) \neq \phi \\ 0 & otherwise \end{cases} \qquad \forall\ y \in Y,\ i = 1,2.$$

ii) If $\underline{B} = (B_1, B_2)$ is an IFS in Y, then the preimage of $\underline{B}$ under f is the IFS in X defined by $f^{-1}(\underline{B}) = (f^{-1}(B_1),\ f^{-1}(B_2))$.

Where $f^{-1}(B_i)(x) = B_i(f(x))\ \forall x \in X,\ i = 1,2$.

Now the intuitionistic fuzzy point introduced by Coker and Demirci in [8] can be modified into the following definition:

Definition1.6

Let X be a nonempty set and let $x \in X$ be a fixed element, α, β $\in I = [0,1]$ such that α ≤ β, β > 0. An intuitionistic fuzzy set, $x_{(\alpha,\beta)} = (x_\alpha,\ x_\beta) \in II^X$ is called intuitionistic fuzzy point (IFP for short) i.e. the IFP $x_{(\alpha,\beta)}$ is an ordered pair of two fuzzy points x_α, x_β where $x_\alpha \leq x_\beta$. The set of all IFPs in X will be denoted by X_{IP}.

An IFP $x_{(\alpha,\beta)}$ is said to be belongs to an IFS $\underline{A} = (A_1, A_2)$ on X, and is denoted by $x_{(\alpha,\beta)} \in \underline{A}$, iff $x_\alpha \in A_1$ and $x_\beta \in A_2$ .The set of all fuzzy points will be denoted by X_f.

Definition 1.7 [13]
Let $A, B \in I^X$. Then A is quasi-coincident with B, iff there exists $x \in X$ such that $A(x)+B(x)>1$. If A is not quasi-coincident with B, then we write $A \not{q} B$, i.e. $A \not{q} B$ iff, $A(x)+B(x) \leq 1$ for all $x \in X$.

Definition 1.8
Let $\underline{A}$, $\underline{B} \in II^X$. Then $\underline{A}$ and $\underline{B}$ are said to be quasi-coincident and it is denoted by $\underline{A} \, q \, \underline{B}$, iff $A_1 \, q \, B_2$ or $A_2 \, q \, B_1$. If $\underline{A}$ is not quasi-coincident with $\underline{B}$, then we denote this by $\underline{A} \not{q} \underline{B}$ i.e. $\underline{A} \not{q} \underline{B} \Leftrightarrow A_1 \not{q} B_2$ and $A_2 \not{q} B_1$.

Theorem 1.9
Let $\underline{A}$, $\underline{B}$, $\underline{C} \in II^X$, $\underline{D}, \underline{F} \in II^Y$, $f: X \to Y$ be a function and $\{ \underline{A}_i : i \in J \} \subseteq II^X$, where $\underline{A}_i = (A_{1i}, A_{2i})$ and $x_{(\alpha,\beta)}$, $y_{(\gamma,\lambda)} \in X_{IP}$. Then:

1) $\underline{A} \not{q} \underline{B} \Leftrightarrow \underline{A} \subseteq \underline{B}^C$,
2) $\underline{A} \cap \underline{B} = \underline{\phi} \Rightarrow \underline{A} \not{q} \underline{B}$,
3) $x_{(\alpha,\beta)} \not{q} \underline{A} \Leftrightarrow x_{(\alpha,\beta)} \in \underline{A}^C$,
4) $\underline{A} \not{q} \underline{A}^C$,
5) $\underline{A} \not{q} \underline{B}$, $\underline{C} \subseteq \underline{B} \Rightarrow \underline{A} \not{q} \underline{C}$,
6) $\underline{A} \subseteq \underline{B} \Leftrightarrow (x_{(\alpha,\beta)} q \, \underline{A} \Rightarrow x_{(\alpha,\beta)} \, q \, \underline{B}$ *for all* $x_{(\alpha,\beta)}$ *in* X),
7) $\underline{A} \, q \, \underline{B} \Leftrightarrow x_{(\alpha,\beta)} \, q \, \underline{B}$, *for some* $x_{(\alpha,\beta)} \in \underline{A}$,
8) $x_{(\alpha,\beta)} \, q \, (\bigcup_{i \in J} \underline{A}_i) \Leftrightarrow \exists \, i \in J$ *such that* $x_{(\alpha,\beta)} \, q \, \underline{A}_i$,
9) *If* $x_{(\alpha,\beta)} \, q \, (\bigcap_{i \in J} \underline{A}_i)$, *then* $x_{(\alpha,\beta)} \, q \, \underline{A}_i$ *for all* $i \in J$,
10) $x \neq y$ *implies* $x_{(\alpha,\beta)} \not{q} y_{(\gamma.\lambda)}$ $\forall \alpha, \beta, \gamma, \lambda \in I$.
11) $x_{(\alpha,\beta)} \not{q} y_{(\gamma.\lambda)} \Leftrightarrow x \neq y$ *or* $x = y$ *and* $\alpha + \lambda \leq 1 \wedge \beta + \gamma \leq 1$.

2. Intuitionistic Fuzzy Topological spaces.

Definition 2.1 [6]
An intuitionistic fuzzy topology (IFT, for short) on a nonempty set X, is a family η of intuitionistic fuzzy sets (IFSs for short) in X that satisfies the following axioms :

T_1) $\underline{\phi}$, $\underline{X} \in \eta$,

T_2) if $\underline{A}, \underline{B} \in \eta$, then $\underline{A} \cap \underline{B} \in \eta$, T3) if $\{ \underline{A}_i : i \in J \} \subseteq \eta$, then $\bigcup_{i \in J} \underline{A}_i \in \eta$.

The pair (X, η) is called intuitionistic fuzzy topological space (IFTS for short). Every element of η is called intuitionistic fuzzy open set (IFOS for short) in X. The complement $\underline{A}^C$ of an IFOS $\underline{A}$ in an IFTS (X, η) is called an intuitionistic fuzzy closed set (IFCS, for short) in X. The set of all IFCSs is denoted by η^C .

Definition 2.2

If (X, η) is an IFTS. An IFS $\underline{N}$ of X is called an intuitionistic fuzzy neighborhood (IFN for short) of an IFP $x_{(\alpha,\beta)}$ in IFTS (X, η) iff there exists IFS $O_{x_{(\alpha,\beta)}} \in \eta$ such that, $x_{(\alpha,\beta)} \subseteq O_{x_{(\alpha,\beta)}} \subseteq \underline{N}$. It is clear that $O_{x_{(\alpha,\beta)}}$ is an IFO- neighborhood (IFON for short) of $x_{(\alpha,\beta)}$. The family of all IF-neighborhoods of the IFP $x_{(\alpha,\beta)}$ will be denoted by **N**($x_{(\alpha,\beta)}$).

Definition 2.3 [6]

Let (X, η) be an IFTS and $\underline{A} = (A_1, A_2)$ be an IFS in X. Then the fuzzy interior and fuzzy closure of $\underline{A}$ will be denoted by $\underline{A}^\circ$, $\overline{\underline{A}}$ respectively, and are defined by:

$\underline{A}^\circ = \bigcup \{ \underline{G} : \underline{G} \textit{ is an IFOS in } X \textit{ and } \underline{G} \subseteq \underline{A} \}$,

$\overline{\underline{A}} = \bigcap \{ \underline{K} : \underline{K} \textit{ is an IFCS in } X \textit{ and } \underline{A} \subseteq \underline{K} \}$,

Proposition 2.4 [6]

For any IFS $\underline{A}$ *in (X,* η *) we have: i)* $\overline{\underline{A}^C} = (\underline{A}^\circ)^C$ *and ii)* $(\underline{A}^C)^\circ = (\overline{\underline{A}})^C$.

Theorem 2.5. *Let (X,* η *) be an IFTS,* $\underline{A}, \underline{B} \in II^X$ *. Then:*

1) $x_{(\alpha,\beta)} \in \underline{A}^\circ \Leftrightarrow \exists\, O_{x_{(\alpha,\beta)}} \in N(x_{(\alpha,\beta)})$ *such that* $O_{x_{(\alpha,\beta)}} \subseteq \underline{A}$,

2) $x_{(\alpha,\beta)}\, q\, \overline{\underline{A}} \Leftrightarrow O_{x_{(\alpha,\beta)}}\, q\, \underline{A} \;\; \forall\, O_{x_{(\alpha,\beta)}} \in N(x_{(\alpha,\beta)})$,

3) $\underline{V}\, q\, \underline{A} \Leftrightarrow \underline{V}\, q\, \overline{\underline{A}} \;\; \forall\, \underline{V} \in \eta$.

Proof. Obvious.

Theorem 2.6. *Every IFTS(X,* η *) generates a fuzzy bitopological space* (X, Π_1, Π_2)*, where* $\Pi_1 = \{ A_1 : \underline{A} \in \eta \}$ *and* $\Pi_2 = \{ A_2 : \underline{A} \in \eta \}$

Theorem 2.7

Let (X, η *) be an IFTS on X. Then the following collections are fuzzy topologies on X,*

i) $\Pi_3 = \{ A_1 : (A_1, X) \in \eta \} \cup \{ \phi \}$

ii) $\Pi_4 = \{ A_2 : (\phi, A_2) \in \eta \} \cup \{ X \}$

iii) $\Pi_\Delta = \{ A : (A, A) \in \eta \}$. *Moreover,* $\Pi_3 \subseteq \Pi_1$, $\Pi_4 \subseteq \Pi_2$ *and* $\Pi_\Delta \subseteq \Pi_1 \cap \Pi_2$.

Proof: Straightforward.

Now we shall introduce a new concept which is the intuitionistic product of two families of fuzzy sets as follows:

Definition2.8

Let $\eta_1, \eta_2 \subseteq I^X$. The intuitionistic product of η_1, η_2 is denoted by $\eta_1 \hat{\times} \eta_2$ and it is defined by $\eta_1 \hat{\times} \eta_2 = \{(A_1, A_2) : A_1 \in \eta_1 , A_2 \in \eta_2 \text{ and } A_1 \subseteq A_2\}$.

Definition. 2.9

An IFTS (X, η) is called, IBTF-topological space iff, $\eta = \Pi_1 \hat{\times} \Pi_2$.

Theorem 2.10

Let (X, τ_1, τ_2) be a fuzzy bitopological space. Then (X, $\tau_1 \hat{\times} \tau_2$) is an IBTF-topological space for which $\Pi_1 = \tau_1$, $\Pi_2 = \tau_2$, where $\tau_1 \hat{\times} \tau_2 = \{ (A_1, A_2) : A_1 \in \tau_1, A_2 \in \tau_2 \text{ and } A_1 \subseteq A_2 \}$.

Proof: Straightforward.

Example 2.11

Let (X, τ) be a fuzzy topological space, $\underline{A} = (A_1, A_2) \in II^X$. Then the following families are IBTF-topologies on X generated by τ :

1) $\eta_1 = \tau \hat{\times} I^X = \{ (A_1, A_2) : A_1 \in \tau \}$

2) $\eta_2 = I^X \hat{\times} \tau = \{ (A_1, A_2) : A_2 \in \tau \}$

3) $\eta_3 = \tau \hat{\times} i(X) = \{ (A_1, X) : A_1 \in \tau \} \cup \{ \underline{\phi} \}$, where $i(X) = (X, \{\underline{0}, \underline{1}\})$ is the indiscrete fuzzy topology on X.

4) $\eta_4 = i(X) \hat{\times} \tau = \{ (\phi, A_2) : A_2 \in \tau \} \cup \{ \underline{X} \}$

5) $\hat{\eta} = \tau \hat{\times} \tau = \{ (A_1, A_2) : A_1, A_2 \in \tau \}$.

Definition 2.12

Let (X, τ) be a fuzzy topological space. Then the IFTS (X, $\tau \hat{\times} \tau$) is called ITF-topological space induced by τ .

Theorem 2.13

Let (X, τ) be a fuzzy topological space and let, $i_\tau : II^X \to II^X$ be an operator defined by $i_\tau(\underline{A}) = (A_1^\circ, A_2^\circ)$ $\forall \underline{A} \in II^X$. Then i_τ is an intuitionistic fuzzy interior operator that generates the ITF-topology $\tau \hat{\times} \tau$.

Proof. It is clear.

Corollary 2.14 *Let (X,τ) be a fuzzy topological space, and let $\underline{A} \in II^X$. Then the map $C_\tau : II^X \to II^X$ defined by $C_\tau(\underline{A}) = (\overline{A_1}, \overline{A_2})$ is an intuitionistic fuzzy closure operator that generates the ITF-topology $\tau \hat{\times} \tau$.*

Theorem2.15

Let (X, τ_1,τ_2) be a fuzzy bitopological space, $\underline{A} \in II^X$. Then the map $C : II^X \Rightarrow II^X$ defined by, $C(\underline{A}) = (\overline{A_1}^2, \overline{\overline{A_1}^2 \cup A_2}^1)$ is an intuitionistic fuzzy closure operator which generates the IBTF-topology $\tau_1 \hat{\times} \tau_2$ on X, where $\overline{A}^i$ is the τ_i -closure of fuzzy set A, $i = 1,2$.

Proof. Obvious.

Corollary 2.16

Let (X, τ_1,τ_2) be a fuzzy bitopological space. Then the map $i : II^X \to II^X$, given by $i(\underline{A}) = \left((A_1 \cap A_2^{\circ 2})^{\circ 1}, A_2^{\circ 2} \right)$ $\forall \underline{A} \in II^X$, is an intuitionistic fuzzy interior operator which generates the IBTF-topology $\tau_1 \hat{\times} \tau_2$ on X, where $A^{\circ i}$ is the τ_i -interior of A, $i = 1,2$.

Definition 2.17 [15]

Let (X,η), (Y,η^*) be IFTSs. Then a map f : X→ Y is said to be

1) An IF-continuous if $f^{-1}(\underline{B})$ is an IFOS of X for all IFOS $\underline{B}$ of Y,
   [or equivalently, $f^{-1}(\underline{B})$ is an IFCS of X for each IFCS $\underline{B}$ of Y],
2) An IF-open function if $f(\underline{A})$ is an IFOS of Y for each IFOS $\underline{A}$ of X,
3) An IF-closed function if $f(\underline{A})$ is an IFCS of Y for each IFCS $\underline{A}$ of X,
4) An IF- homeomorphism if f is bijective and f, f^{-1} are IF-continuous.

Definition 2.18 [15]

Let $f : (X,\eta) \to (Y,\eta^*)$ be a map. Then the following statements are equivalent:

i) f is an IF-continuous map.

ii) $\forall\ x_{(\alpha,\beta)} \in X_{IP}$, $\forall$ IFN $O_{f(x_{(\alpha,\beta)})}$ of $f(x_{(\alpha,\beta)})$ there is an IFN $O_{x_{(\alpha,\beta)}}$ of $x_{(\alpha,\beta)}$ such that $f(O_{x_{(\alpha,\phi)}}) \subseteq O_{f(x_{(\alpha,\beta)})}$.

Theorem 2.19 [15]

Let $f : (X,\eta) \to (Y,\eta^*)$ be a bijection. Then the following statements are equivalent:

i) f is an IF-homeomorphism .

ii) f is an IF- continuous and IF-closed .

iii) $\overline{f^{-1}(\underline{B})} = f^{-1}(\overline{\underline{B}})$ $\forall$ IFS $\underline{B}$ of Y.

Theorem 2.20

Let $f:(X,\eta)\to(Y,\eta^*)$ *be an IF-continuous function. Then the function:* $f:(X,\Pi_i)\to(Y,\Pi_i^*),\ i=1,2$ *are fuzzy continuous functions, where* Π_1, Π_2 *are fuzzy topologies defined as in theorem(2.6).*

Proof.

Let $f:(X,\eta)\to(Y,\eta^*)$ be an IF-continuous function and let $B_1\in\Pi_1^*$. Then there exists $B_2\in\Pi_2^*$ such that $(B_1,B_2)\in\eta^*$ and hence, $f^{-1}(B_1,B_2)=(f^{-1}(B_1),f^{-1}(B_2))\in\eta$, consequently, $f^{-1}(B_1)\in\Pi_1$, which implies that $f:(X,\Pi_1)\to(Y,\Pi_1^*)$ is a fuzzy continuous.

The following example shows that the converse of the above theorem may not be true in general.

Example 2.21 Let (X,τ) be any fuzzy topological space. Then $id:(X,\tau)\to(X,\tau)$ is a fuzzy continuous . But $id:(X,\eta_\Delta)\to(X,\tau\hat{\times}\tau)$, is not IF-continuous. Where $\eta_\Delta=\{(A,A):A\in\tau\}$.

Theorem 2.22 *Let* (X,η) *be an IBTF-topological space and* (Y,η^*) *be any IFTS. Then* $f:(X,\eta)\to(Y,\eta^*)$ *is an IF-continuous function iff* $f:(X,\Pi_i)\to(Y,\Pi_i^*)$, $i=1,2$ *are fuzzy continuous functions.*

Proof. it is clear.

3. Separation axioms in intuitionistic fuzzy topological spaces.

Definition 3.1

An intuitionistic fuzzy topological space (X,η) is said to be:

1) IF-$T_\circ$ iff $x_{(\alpha,\beta)}\ q\!\!\!/\ y_{(\gamma,\lambda)}$ implies $x_{(\alpha,\beta)}\ q\!\!\!/\ \bar{y}_{(\gamma,\lambda)}$ or $y_{(\gamma,\lambda)}\ q\!\!\!/\ \bar{x}_{(\alpha,\beta)}$.
2) IF-T_1 iff $x_{(\alpha,\beta)}\ q\!\!\!/\ y_{(\gamma,\lambda)}$ implies $x_{(\alpha,\beta)}\ q\!\!\!/\ \bar{y}_{(\gamma,\lambda)}$ and $y_{(\gamma,\lambda)}\ q\!\!\!/\ \bar{x}_{(\alpha,\beta)}$.
3) IF-T_2 iff $x_{(\alpha,\beta)}\ q\!\!\!/\ y_{(\gamma,\lambda)}$ implies there exists $O_{x(\alpha,\beta)}$, $O_{y(\gamma,\lambda)}$ such that $O_{x(\alpha,\beta)}\ q\!\!\!/\ O_{y(\gamma,\lambda)}$.

Now we list some characterizations of IF-separation axioms.

Theorem 3.2

Let (X,η) *be an IFTS. Then* (X,η) *is IF-*T_0 *iff [* $x_{(\alpha,\beta)}\ q\!\!\!/$ *, there exists* $O_{x(\alpha,\beta)}$ *such that* $y_{(\gamma,\lambda)}\ q\!\!\!/\ O_{x(\alpha,\beta)}$ *or there exists* $O_{y(\gamma,\lambda)}$ *such that* $x_{(\alpha,\beta)}\ q\!\!\!/\ O_{y(\gamma,\lambda)}\ \forall\ x_{(\alpha,\beta)},\ y_{(\gamma,\lambda)}\in X_{IP}$*].*

Proof. Follows from (2) of theorem (2.5) and (5) of theorem (1.9).

Theorem 3.3 *Let (X, η) be an IFTS. Then the following statements are equivalent:*

i) (X, η) $\in$ IF-T_1

ii) $x_{(\alpha,\beta)} q\!\!\!/\, y_{(\gamma,\lambda)}$ implies that there exists $O_{x(\alpha,\beta)}$ such that $y_{(\gamma,\lambda)} q\!\!\!/\, O_{x(\alpha,\beta)}$ and there exists $O_{y(\gamma,\lambda)}$ such that $x_{(\alpha,\beta)} q\!\!\!/\, O_{y(\gamma,\lambda)}$,

iii) $\bar{x}_{(\alpha,\beta)} = x_{(\alpha,\beta)}$ $\forall\ x_{(\alpha,\beta)} \in X_{IP}$.

Proof. i) $\Rightarrow$ ii) is obvious.

i) $\Rightarrow$iii) Let $x_{(\alpha,\beta)} q\!\!\!/\, y_{(\gamma,\lambda)}$. Then by (ii) there exists $O_{y(\gamma,\lambda)}$ such that $x_{(\alpha,\beta)} q\!\!\!/\, O_{y(\gamma,\lambda)}$ this implies $O_{y(\gamma,\lambda)} \subseteq x^{C}_{(\alpha,\beta)}$, thus $x^{C}_{(\alpha,\beta)}$ is open for every $x_{(\alpha,\beta)} \in X_{IP}$ i.e. $x_{(\alpha,\beta)}$ is closed for every $x_{(\alpha,\beta)} \in X_{IP}$ hence $\bar{x}_{(\alpha,\beta)} = x_{(\alpha,\beta)}$.

iii) $\Rightarrow$i) Let $\bar{x}_{(\alpha,\beta)} = x_{(\alpha,\beta)}$ $\forall\ x_{(\alpha,\beta)} \in X_{IP}$ and $x_{(\alpha,\beta)} q\!\!\!/\, y_{(\gamma,\lambda)}$. Then $x_{(\alpha,\beta)}$, $y_{(\gamma,\lambda)}$ are closed IFPs. Since $y_{(\gamma,\lambda)} q\!\!\!/\, y^{C}{}_{(\gamma,\lambda)} = O_{x(\alpha,\beta)}$ and $x_{(\alpha,\beta)} q\!\!\!/\, x^{C}{}_{(\alpha,\beta)} = O_{y(\gamma,\lambda)}$, then $x_{(\alpha,\beta)} q\!\!\!/\, \bar{y}_{(\gamma,\lambda)}$ and $y_{(\gamma,\lambda)} q\!\!\!/\, \bar{x}_{(\alpha,\beta)}$.

Hence (X, η) is IF- T_1.

Theorem 3.4 *If (X, η) is IF-T_2 then* $x_{(\alpha,\beta)} = \bigcap_{O_{x(\alpha,\beta)} \in N(x_{(\alpha,\beta)})} \overline{O}_{x(\alpha,\beta)}$ $\forall\ x_{(\alpha,\beta)} \in X_{IP}$.

Proof. Let (X, η) be an IF-T_2 and $x_{(\alpha,\beta)} \in X_{IP}$. Then for any $y_{(\gamma,\lambda)} q\!\!\!/\, x_{(\alpha,\beta)}$ there exists $O_{y(\gamma,\lambda)} \in N(y_{(\gamma,\lambda)})$, $O_{x(\alpha,\beta)} \in N(x_{(\alpha,\beta)})$ such that $O_{y(\gamma,\lambda)} q\!\!\!/\, O_{x(\alpha,\beta)} \Rightarrow y_{(\gamma,\lambda)} q\!\!\!/\, \overline{O}_{x(\alpha,\beta)} \Rightarrow y_{(\gamma,\lambda)} q\!\!\!/ \bigcap_{O_{x(\alpha,\beta)} \in N(x_{(\alpha,\beta)})} \overline{O}_{x(\alpha,\beta)}$

$\Rightarrow x_{(\alpha,\beta)} \supseteq \bigcap_{O_{x(\alpha,\beta)} \in N(x_{(\alpha,\beta)})} \overline{O}_{x(\alpha,\beta)}$ (by (6) of theorem (1.9)). On other hand it is clearly that $x_{(\alpha,\beta)} \subseteq \bigcap_{O_{x(\alpha,\beta)} \in N(x_{(\alpha,\beta)})} \overline{O}_{x(\alpha,\beta)}$, and so $x_{(\alpha,\beta)} = \bigcap_{O_{x(\alpha,\beta)} \in N(x_{(\alpha,\beta)})} \overline{O}_{x(\alpha,\beta)}$.

Definition 3.5 An IF-topological space (X, η) is said to be:

1) IF-$R_{\circ}$ iff $x_{(\alpha,\beta)} q\!\!\!/\, \bar{y}_{(\gamma,\lambda)}$ implies $y_{(\gamma,\lambda)} q\!\!\!/\, \bar{x}_{(\alpha,\beta)}$.

2) IF-R_1 iff $x_{(\alpha,\beta)} q\!\!\!/\, \bar{y}_{(\gamma,\lambda)}$ implies there exists $O_{x(\alpha,\beta)}$, $O_{y(\gamma,\lambda)}$ such that, $O_{x(\alpha,\beta)} q\!\!\!/\, O_{y(\gamma,\lambda)}$.

3) IF-R_2 iff $x_{(\alpha,\beta)} q\!\!\!/\, \underline{F}$, $\underline{F} \in \eta^{C}$ implies there exists $O_{x(\alpha,\beta)}$, $O_{\underline{F}}$ such that $O_{x(\alpha,\beta)} q\!\!\!/\, O_{\underline{F}}$.

4) IF-R_3 iff $\underline{F}_1 q\!\!\!/\, \underline{F}_2$, $\underline{F}_1$, $\underline{F}_2 \in \eta^{C}$ implies there exists $O_{\underline{F}_1}$, $O_{\underline{F}_2}$ such that $O_{\underline{F}_1} q\!\!\!/\, O_{\underline{F}_2}$.

5) IF-T_3 iff it is IF-R_2 and IF-T_1.
6) IF-T_4 iff it is IF-R_3 and IF-T_1.

Now we introduce the following reformulation of IF-R_i *axioms*, $(i = 0, 1, 2, 3)$ and give some implications on it.

Theorem 3.6

Let (X,η) be an IFTS, $x_{(\alpha,\beta)} \in X_{IP}$. Then the following statements are equivalent:

1) (X,η) is an IF-$R_\circ$.

2) $\vec{x}_{(\alpha,\beta)} \subseteq O_{x(\alpha,\beta)} \quad \forall\ O_{x(\alpha,\beta)} \in N(x_{(\alpha,\beta)})$.

3) $\vec{x}_{(\alpha,\beta)} \subseteq \bigcap \{ O_{x(\alpha,\beta)} : O_{x(\alpha,\beta)} \in N(x_{(\alpha,\beta)}) \}$.

4) $x_{(\alpha,\beta)}\ \not q\ \underline{F}$, $\underline{F} \in \eta^C$ implies there exists $O_{\underline{F}}$ such that $x_{(\alpha,\beta)}\ \not q\ O_{\underline{F}}$.

5) $x_{(\alpha,\beta)}\ \not q\ \underline{F}$, $\underline{F} \in \eta^C$ implies $\bar{x}_{(\alpha,\beta)}\ \not q\ \underline{F}$.

6) $x_{(\alpha,\beta)}\ \not q\ \bar{y}_{(\gamma,\lambda)}$ implies $\bar{x}_{(\alpha,\beta)}\ \not q\ \bar{y}_{(\gamma,\lambda)}$.

Proof. 1)$\Rightarrow$2) Let $y_{(\gamma,\lambda)}\, q\, \vec{x}_{(\alpha,\beta)} \overset{(1)}{\Rightarrow} x_{(\alpha,\beta)}\, q\, \bar{y}_{(\gamma,\lambda)}$. By (2) of theorem (2.5), we have $y_{(\gamma,\lambda)}\, q\, O_{x(\alpha,\beta)}$, $\forall\ O_{x(\alpha,\beta)} \Rightarrow \vec{x}_{(\alpha,\beta)} \subseteq O_{x(\alpha,\beta)}\ \forall\ O_{x(\alpha,\beta)}$ (by (6) of theorem 1.9).

2) $\Rightarrow$3) is obvious.

3) $\Rightarrow$ 4) Let $x_{(\alpha,\beta)}\ \not q\ \underline{F}$, $\underline{F} \in \eta^C \Rightarrow x_{(\alpha,\beta)} \in \underline{F}^C \overset{(2)}{\Rightarrow} \vec{x}_{(\alpha,\beta)} \subseteq \underline{F}^C \Rightarrow \underline{F} \subseteq \vec{x}^C{}_{(\alpha,\beta)} = O_{\underline{F}}$, and hence $x_{(\alpha,\beta)}\ \not q\ \vec{x}^C{}_{(\alpha,\beta)} = O_{\underline{F}}$.

4) $\Rightarrow$5) Let $x_{(\alpha,\beta)}\, \not q\, \underline{F}$, $\underline{F} \in \eta^C \overset{(4)}{\Rightarrow}$ there exists $O_{\underline{F}}$, such that

$x_{(\alpha,\beta)}\, \not q\, O_{\underline{F}} = x_{(\alpha,\beta)}\, \not q\, O_{\underline{F}} \Rightarrow x_{(\alpha,\beta)} \in O_{\underline{F}}{}^C \Rightarrow \vec{x}_{(\alpha,\beta)} \subseteq O_{\underline{F}}^C \Rightarrow \vec{x}_{(\alpha,\beta)}\, \not q\, O_{\underline{F}} \Rightarrow \vec{x}_{(\alpha,\beta)}\, \not q\, \underline{F}$.

5) $\Rightarrow$6) and 6) $\Rightarrow$1) are obvious.

Theorem 3.7 *The following implications hold:*

IF-$R_3 \wedge$ IF-$R_0 \Rightarrow$IF-$R_2 \Rightarrow$IF-$R_1 \Rightarrow$IF-R_0.

Proof. It is clear.

Theorem 3.8 *The following implications hold:*

1) IF-$T_4 \Rightarrow$IF-$T_3 \Rightarrow$IF-$T_2 \Rightarrow$IF-$T_1 \Rightarrow$IF-T_0.

2) IF-$R_2 \wedge$IF-$T_0 \Rightarrow(X, \eta)$ is IF-T_3

Proof. It is clear

Theorem 3.9 *Let (X,η) be an IFTS. Then (X,η) is IF-R_1 iff $x_{(\alpha,\beta)} \not q$ $\bar{y}_{(\gamma,\lambda)}$ implies there exist $O_{\bar{x}(\alpha,\beta)}$ and $O_{\bar{y}(\gamma,\lambda)}$ such that $O_{\bar{x}(\alpha,\beta)} \not q\, O_{\bar{y}(\gamma,\lambda)}$.*

Proof. Follows from the last implication of theorem (3.7) and from (2) of theorem(3.6).

Theorem 3.10

Let (X,η) be an IFTS. Then (X,η) is IF-R_2 iff $\forall\ x_{(\alpha,\beta)} \in X_{IP}$, $\forall\ O_{x_{(\alpha,\beta)}} \in N(x_{(\alpha,\beta)})$ there exists $O^*_{x_{(\alpha,\beta)}}$ such that $\overline{O}^*_{x_{(\alpha,\beta)}} \subseteq O_{x_{(\alpha,\beta)}}$.

Proof. Let (X,η) be an IF-R_2, $x_{(\alpha,\beta)} \in X_{IP}$, $O_{x_{(\alpha,\beta)}} \in N(x_{(\alpha,\beta)})$. Then $x_{(\alpha,\beta)} \not q\, O^C_{x_{(\alpha,\beta)}}$ implies that $\exists\ O^*_{x_{(\alpha,\beta)}} \in N(x_{(\alpha,\beta)})$, $\underline{V} \in N(O^C_{x_{(\alpha,\beta)}})$ such that $O^*_{x_{(\alpha,\beta)}} \not q\, \underline{V} \Rightarrow O^*_{x_{(\alpha,\beta)}} \subseteq \underline{V}^C \Rightarrow \overline{O}^*_{x_{(\alpha,\beta)}} \subseteq \underline{V}^C \subseteq O_{x(\alpha,\beta)}$.

Conversely, let $x_{(\alpha,\beta)} \in X_{IP}$, $\underline{F} \in \eta^C$ be such that $x_{(\alpha,\beta)} \not q\, \underline{F}$.

Then $x_{(\alpha,\beta)} \subseteq \underline{F}^C$ so, $\underline{F}^C \in N(x_{(\alpha,\beta)}) \Rightarrow \exists\ O^*_{x_{(\alpha,\beta)}}$ such that $\overline{O}^*_{x_{(\alpha,\beta)}} \subseteq O_{x(\alpha,\beta)} = \underline{F}^C$ (by hypothesis), hence $\underline{F} \subseteq \overline{O}^{*C}_{x_{(\alpha,\beta)}} = O_{\underline{F}}$ and $O_{\underline{F}} \not q\, O^*_{x_{(\alpha,\beta)}} \Rightarrow (X,\eta) \in$ IF-R_2.

Theorem 3.11

Let (X,η) be an IFTS. Then $(X,\eta) \in$ IF-R_3 iff $\forall\ \underline{F} \in \eta^C$, $\forall\ O_{\underline{F}} \in N(\underline{F})$ there exists $O_{\underline{F}}^* \in N(\underline{F})$ such that $\overline{O_{\underline{F}}^*} \subseteq O_{\underline{F}}$.

Proof. It is similar to that of the above theorem.

Theorem3.12

Let (X,η) be an IFTS. Then the following implications hold:

$$\begin{array}{ccccccccc} IF\text{-}T_4 & \Rightarrow & IF\text{-}T_3 & \Rightarrow & IF\text{-}T_2 & \Rightarrow & IF\text{-}T_1 & \Rightarrow & IF\text{-}T_0 \\ \Downarrow & & \Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\ IF\text{-}R_3 \wedge IF\text{-}R_0 & \Rightarrow & IF\text{-}R_2 & \Rightarrow & IF\text{-}R_1 & \Rightarrow & IF\text{-}R_0 & \Rightarrow & \text{any IF-space.} \end{array}$$

Proof. Straightforward.

Definition 3.13

An intuitionistic fuzzy topological property (IFTP for short) for an IFTS (X,η), will be called good extension property if, (X,τ) has the fuzzy topological property (FTP for short) iff $(X,\tau \hat{\times} \tau)$ has the fuzzy topological property IFTP.

Now we shall show that all intuitionistic fuzzy regularity axioms ($IF-R_i$, $i = 0,1,2,3$) are good extensions properties.

Theorem3.14 *The IF- regularity axioms(IF-* $R_i, i = 0,1,2,3$*) are good extension properties.*

Proof. As a sample we prove the cases ($i = 2$) the remaining cases are similar.

For $i = 2$. Let (X, τ)be a FR$_2$-space, $x_{(\alpha,\beta)}\, q\!\!\!/\ \underline{F}$, $\underline{F}$ = (F_1,F_2)$\in \tau^C \hat{\times} \tau^C$. Then $x_\alpha\, q\!\!\!/\ F_2$ and $x_\beta\, q\!\!\!/\ F_1$ where F_1, $F_2 \in \tau^C \Rightarrow \exists\ O_{x_\alpha}, O_{F_2}$, O_{x_β} and O_{F_1} s.t $O_{x_\alpha}\, q\!\!\!/\ O_{F_2}$ and $O_{x_\beta}\, q\!\!\!/\ O_{F_1}$ .Take $O^*_{x_\alpha} = O_{x_\alpha} \bigcap O_{x_\beta}$ and $O^*_{F_1} = O_{F_1} \bigcap O_{F_2}$. Then $O_{x(\alpha,\beta)} = (O^*_{x_\alpha}, O_{x_\beta})$ and, $O_{\underline{F}} = (O_{F_1}{}^*, O_{F_2})$ such that $O_{x_{(\alpha,\beta)}}\, q\!\!\!/\ O_{\underline{F}} \Rightarrow$ (X, $\tau \hat{\times} \tau$) is IF-R$_2$.

Conversely, let (X, $\tau \hat{\times} \tau$) be an IF-R$_2$ space, $x_\alpha\, q\!\!\!/\ F$, $F \in \tau^C$. Then

$(x_\alpha, x_\alpha) q\!\!\!/\ (F, F)$ i.e. $x_{(\alpha,\alpha)}\, q\!\!\!/\ \underline{F} = (F, F) \in \tau^C \hat{\times} \tau^C \Rightarrow$

$\exists\ O_{x(\alpha,\alpha)} = (O_{x_\alpha}, O_{x_\alpha}), O_{\underline{F}} = (O_F, O_F)$ such that $O_{x(\alpha,\alpha)}\, q\!\!\!/\ O_{\underline{F}} \Rightarrow$

$\exists\ O_{x_\alpha}, O_F$ such that $O_{x_\alpha}\, q\!\!\!/\ O_F$, hence (X, τ) is FR$_2$.

Now we shall show that the separation axioms($IF - T_i$, $i = 1, 2, 3, 4$) are good extensions.

Theorem 3.15

The IF- separation axioms (IF-$T_i, i = 1,2,3,4$*) are good extension Properties.*

Proof. As a sample we prove the cases ($i = 1,3$) the remaining cases are similar.

i) For $i = 1$. Let (X, τ) be a FT$_1$-space and let $x_{(\alpha,\beta)}$ be an IFP of (X, $\tau \hat{\times} \tau$). Then $\bar{x}_{(\alpha,\beta)} = (\bar{x}_\alpha, \bar{x}_\beta) = (x_\alpha, x_\beta) = x_{(\alpha,\beta)}$, (since (X, τ)$\in$FT$_1$). Thus (X, $\tau \hat{\times} \tau$) is IF-T$_1$.

Conversely, Let (X, $\tau \hat{\times} \tau$) be an IF-T$_1$ space, x_α be a fuzzy point of (X, τ). Then $\bar{x}_{(\alpha,\alpha)} = (\bar{x}_\alpha, \bar{x}_\alpha) = x_{(\alpha,\alpha)} = (x_\alpha, x_\alpha)$ i.e. $\bar{x}_\alpha = x_\alpha$. Hence (X, τ) is FT$_1$-space.

ii) For $i = 3$. The proof follows from ii) of the above theorem and from i).

Theorem 3.16

Let (X, τ *) be a fuzzy topological space. Then(X,* $\tau \hat{\times} \tau$ *) is IF-T*$_0$ $\Rightarrow$ *(X,* τ *) is FT*$_0$*-space.*

Proof. Let (X, $\tau \hat{\times} \tau$) be an IF-T$_0$ space, $x_\alpha\, q\!\!\!/\ y_\beta$. Then

$(x_\alpha, x_\alpha) q\!\!\!/ (y_\beta, y_\beta)$ i.e. $x_{(\alpha,\alpha)} q\!\!\!/ y_{(\beta,\beta)} \Rightarrow x_{(\alpha,\alpha)} q\!\!\!/ \bar{y}_{(\beta,\beta)}$ or $y_{(\beta,\beta)} q\!\!\!/ \bar{x}_{(\alpha,\alpha)}$ (since(X, $\tau \hat{\times} \tau$) $\in$ IF-T_0) $\Rightarrow x_\alpha q\!\!\!/ \bar{y}_\beta$ or $y_\beta q\!\!\!/ \bar{x}_\alpha$, and so (X,τ) is *IF-$T_0$* .

The following example shows that the converse of the above theorem may not be true.

Example 3.17 *Let $X = \{a\}$, and $\tau = \{a_t : t \le 1/2\} \cup \{X\}$ be a fuzzy topology on X . Then (X , τ) is FT_0-space. But the intuitionistic fuzzy topological spaces ($X \tau \hat{\times} \tau$), where*
$\tau \hat{\times} \tau = \{(a_t, X) : t \le 1/2\} \cup \{(a_t, a_r) : t \le r \le 1/2\} \cup \{\underline{X}\}$ *is not IF-T_0* .

Definition 3.18 A property p is called an intuitionistic fuzzy topological property or intuitionistic fuzzy topological invariant if an intuitionistic fuzzy topological space (X, η) has p then every space homeomorphic to space (X, η) has also p.

Theorem 3.19

The regularity axioms(IF-R_i , $i = 0,1,2,3$)are intuitionistic fuzzy topological properties.

Proof: As a sample we prove the cases $i = 2$.

.For $i = 2$. Let (X, η) be an $IF - R_2$ and let $f : (X, \eta) \to (Y, \eta^*)$ be an IF-homeomorphism. Let $x_{(\alpha,\beta)}$ are IFP in Y and $\underline{F} \in \eta^{*C}$ such that $x_{(\alpha,\beta)} q\!\!\!/ \underline{F}$, then $f^{-1}(x_{(\alpha,\beta)}) q\!\!\!/ f^{-1}(\underline{F})$, $f^{-1}(\underline{F}) \in \eta^C$. Since (X, η) is $IF - R_2$. Then there exists $O_{f^{-1}(x_{(\alpha,\beta)})}$ and $O_{f^{-1}(\underline{F})}$ Such that $O_{f^{-1}(x_{(\alpha,\beta)})} q\!\!\!/ O_{f^{-1}(\underline{F})}$. Now put, $O_{x(\alpha,\beta)} = \left(f(O^C_{f^{-1}(x_{(\alpha,\beta)})})\right)^C \in \eta^*$,
$O_{\underline{H}} = \left(f(O^C_{f^{-1}(\underline{H})})\right)^C \in \eta^*$. Then there exists $O_{x(\alpha,\beta)}$ *and* $O_{\underline{H}} \in \eta^*$ such that $O_{x(\alpha,\beta)} q\!\!\!/ O_{\underline{H}}$. Hence (Y, η^*) is $IF - R_2$.

Theorem 3.20

The separation axioms IF-T_i , $i = 0,1,2,3,4$ are intuitionistic fuzzy topological properties.

Proof. As a sample we prove the cases $i = 1$.

For $i = 1$. Let (X, η) be an $IF - T_1$ and let $f : (X, \eta) \to (Y, \eta^*)$ be an IF-homeomorphism . Let $x_{(\alpha,\beta)}$, $y_{(\gamma,\lambda)}$ are IFPs in Y such that $x_{(\alpha,\beta)} q\!\!\!/ y_{(\gamma,\lambda)}$. Then $f^{-1}(x_{(\alpha,\beta)}) q\!\!\!/ f^{-1}(y_{(\gamma,\lambda)})$. Since (X, η) is $IF - T_1$. Then
$f^{-1}(x_{(\alpha,\beta)}) q\!\!\!/ \overline{f^{-1}(y_{(\gamma,\lambda)})}$ and $f^{-1}(y_{(\gamma,\lambda)}) q\!\!\!/ \overline{f^{-1}(x_{(\alpha,\beta)})} \Rightarrow$

$f^{-1}(x_{(\alpha,\beta)})\, q\!\!\!/\, f^{-1}(\bar{y}_{(\gamma,\lambda)})$ and $f^{-1}(y_{(\gamma,\lambda)})\, q\!\!\!/\, f^{-1}(\bar{x}_{(\alpha,\beta)})$ (by theorem (2.20)) $\Rightarrow$ $f\, f^{-1}(x_{(\alpha,\beta)})\, q\!\!\!/\, f\, f^{-1}(\bar{y}_{(\gamma,\lambda)})$ and $f\, f^{-1}(y_{(\gamma,\lambda)})\, q\!\!\!/\, f\, f^{-1}(\bar{x}_{(\alpha,\beta)})$ $\Rightarrow$ $x_{(\alpha,\beta)}\, q\!\!\!/\, \bar{y}_{(\gamma,\lambda)}$ and $y_{(\gamma,\lambda)}\, q\!\!\!/\, \bar{x}_{(\alpha,\beta)} \Rightarrow (Y,\eta^*)\ is\ IF-T_1$.

Theorem 3.21

The axioms IF-R_3 and IF-T_i , $i=1,4$ are invariant under IF- continuous and IF-closed onto map.

Proof. i) Let (X, η) be an $IF-R_3$ and let $f:(X,\eta)\to(Y,\eta^*)$ be an IF-continuous, IF-closed and onto map. Let $\underline{M},\ \underline{G}\in\eta^{*C}$ such that $\underline{M}\ q\!\!\!/\ \underline{G}$. Then $f^{-1}[\underline{M}]\ q\!\!\!/\ f^{-1}[\underline{G}]$ and $f^{-1}[\underline{M}],\ f^{-1}[\underline{G}]\in\eta^C$.

Since $(X,\eta)\ is\ an\, IF-R_3$, then there exists $O_{f^{-1}[\underline{M}]}$, $O_{f^{-1}[\underline{G}]}\in\eta$ such that $O_{f^{-1}[\underline{M}]}\ q\!\!\!/\ O_{f^{-1}[\underline{G}]}$. Let $\underline{U}=\left(f[O^C_{f^{-1}[\underline{M}]}]\right)^C$ and $\underline{V}=\left(f[O^C_{f^{-1}[\underline{G}]}]\right)^C$. Then $\underline{M}\subseteq\underline{U}\in\eta^*$, $\underline{G}\subseteq\underline{V}\in\eta^*$ and $\underline{U}\ q\!\!\!/\ \underline{V}$, Then $(Y,\eta^*)\ is\ IF-R_3$.

ii) For $i=1$. Let (X, η) be an $IF-T_1$, $f:(X,\eta)\to(Y,\eta^*)$ be an IF-continuous IF-closed and onto map. Let $x_{(\alpha,\beta)}$, $y_{(\gamma,\lambda)}$ are IFPs in Y such that $x_{(\alpha,\beta)}\ q\!\!\!/\ y_{(\gamma,\lambda)}$ .Then $f^{-1}(x_{(\alpha,\beta)})\ q\!\!\!/\ f^{-1}(y_{(\gamma,\lambda)})$. Since (X, η) is $IF-T_1$, then there exists $O_{f^{-1}(x_{(\alpha,\beta)})}$ and $O_{f^{-1}(y_{(\gamma,\lambda)})}\in\eta$ such that $O_{f^{-1}(x_{(\alpha,\beta)})}\ q\!\!\!/\ f^{-1}(y_{(\gamma,\lambda)})$ and $f^{-1}(x_{(\alpha,\beta)})\ q\!\!\!/\ O_{f^{-1}(y_{(\gamma,\lambda)})}$. Now take $\underline{U}=\left(f(O^C_{f^{-1}(x_{(\alpha,\beta)})})\right)^C$ and $\underline{V}=\left(f(O^C_{f^{-1}(y_{(\gamma,\lambda)})})\right)^C$ then there exists $O_{x_{(\alpha,\beta)}}=\underline{U}\in\eta^*$ and $O_{y_{(\gamma,\lambda)}}=\underline{V}\in\eta^*$ (since $f\ is\ IF-closed$) such that $f^{-1}(O_{x_{(\alpha,\beta)}})\subseteq O_{f^{-1}(x_{(\alpha,\beta)})}$ and $f^{-1}(O_{y_{(\gamma,\lambda)}})\subseteq O_{f^{-1}(y_{(\gamma,\lambda)})}\Rightarrow f^{-1}(O_{x_{(\alpha,\beta)}})\ q\!\!\!/\ f^{-1}(y_{(\gamma,\lambda)})$ and $f^{-1}(x_{(\alpha,\beta)})\ q\!\!\!/\ f^{-1}(O_{y_{(\gamma,\lambda)}})\Rightarrow f^{-1}(O_{x_{(\alpha,\beta)}})\subseteq f^{-1}(y^C_{(\gamma,\lambda)})$ and $f^{-1}(O_{y_{(\gamma,\lambda)}})\subseteq f^{-1}(x^C_{(\alpha,\beta)})\Rightarrow f\, f^{-1}(O_{x_{(\alpha,\beta)}})\subseteq f\, f^{-1}(y^C_{(\gamma,\lambda)})$ and $f\, f^{-1}(O_{y_{(\gamma,\lambda)}})\subseteq f\, f^{-1}(x^C_{(\alpha,\beta)})\Rightarrow O_{x_{(\alpha,\beta)}}\subseteq y^C_{(\gamma,\lambda)}$ and $O_{y_{(\gamma,\lambda)}}\subseteq x^C_{(\alpha,\beta)}$ $\Rightarrow O_{x_{(\alpha,\beta)}}\ q\!\!\!/\ y_{(\gamma,\lambda)}$ and $x_{(\alpha,\beta)}\ q\!\!\!/\ O_{y_{(\gamma,\lambda)}}$ (by (1),(5) of theorem (1.9)).

Hence from (ii) of theorem (3.3) we obtain $(Y,\eta^*)\ is\ IF-T_1$.

iii) For $i=4$. The proof follows from i) and ii).

4. The Relations between fuzzy separation axioms of IFTSs and that of induced fuzzy topological spaces:

Theorem 4.1 *If the IFTS* (X,η) *is IF-*T_0 *, then* (X,Π_1) *,* (X, Π_2) *are* FT_0*.*
Proof. Let (X, η) be an IF-T_0 , $x_\alpha \not q\, y_\beta$. Then $(x_\alpha, x_\alpha) \not q\, (y_\beta, y_\beta)$
i.e. $x_{(\alpha,\alpha)} \not q\, y_{(\beta,\beta)} \Rightarrow \exists$ IFOS $O_{x(\alpha,\alpha)} = (A_1, A_2) \in N(x_{(\alpha,\alpha)})$ such that $y_{(\beta,\beta)} \not q\, O_{x(\alpha,\alpha)} = (A_1, A_2) \Rightarrow y_\beta \not q\, O_{x_\alpha} = A_1 \in \Pi_1$ or there exists an IFOS $O_{y(\beta,\beta)} = (B_1, B_2) \in N(y_{(\beta,\beta)})$ such that $x_{(\alpha,\alpha)} \not q\, O_{y(\beta,\beta)} = (B_1, B_2)$
$\Rightarrow x_\alpha \not q\, B_1 = O_{y_\beta} \in \Pi_1 \Rightarrow (X, \Pi_1)$ is FT_0. The rest is similar.
The converse of the above theorem may not be true in general this can be shown by an example (3.18), where $\tau = \Pi_1 = \Pi_2$.
Theorem 4.2 *If the IFTS* (X,η) *is IF-*T_1 *, then* (X,Π_1) *,* (X,Π_2) *are* FT_1*.*

Proof. Let (X, η) be an IF-T_1, $x_\alpha \not q\, y_\beta \Rightarrow (x_\alpha, x_\alpha) \not q\, (y_\beta, y_\beta)$ i.e. $x_{(\alpha,\alpha)} \not q\, y_{(\beta,\beta)} \Rightarrow$ there exists an IFOS $O_{x(\alpha,\alpha)} = (A_1, A_2) \in N(x_{(\alpha,\alpha)})$ such that $y_{(\beta,\beta)} \not q\, (A_1, A_2) \Rightarrow y_\beta \not q\, A_1 = O_{x_\alpha} \in \Pi_1$ and there exists an IFOS $O_{y(\beta,\beta)} = (B_1, B_2) \in N(y_{(\beta,\beta)})$ such that $x_{(\alpha,\alpha)} \not q\, O_{y(\beta,\beta)} = (B_1, B_2) \Rightarrow x_\alpha \not q\, B_1 = O_{y_\beta} \in \Pi_1$ and hence (X, Π_1) is FT_1-space.The rest is similar.
The following example shows that the converse of the above theorem may not be true.
Example 4.3 *Let* $X = \{a\}$*, and* $\eta = \{(a_t, a_r) : t \le r \le 1/2\} \cup \{(a_t, a_r) : t = r > 1/2\}$ *be an IFT on X. Then* (X,η) *is not IF-*T_1*-topological space. But the induced fuzzy topological spaces* (X,Π_i), $i = 1,2$ *, where* $\Pi_1 = \Pi_2 = \{a_t : t \in I = [0,1]\}$ *are* FT_1*-spaces.*
Theorem 4.4
For any pair of fuzzy topological spaces (X, τ_i), $i = 1,2$. *Then* $(X, \tau_1 \hat{\times} \tau_2)$ *is IF-*T_1 *iff,* (X, τ_i), $i = 1,2$ *are* FT_1*-topological spaces.*
Proof. It is clear.
Theorem4.5 *If the IFTS* (X,η) *is IF-*T_2 *, then* (X,Π_1) *is* FT_2*.*
Proof. Let (X, η) be an IF-T_2, $x_\alpha \not q\, y_\beta$. Then $x_{(\alpha,\alpha)} \not q\, y_{(\beta,\beta)} \Rightarrow \exists$ IFOS $O_{x(\alpha,\alpha)} = (A_1, A_2)$ and IFOS $O_{y(\beta,\beta)} = (B_1, B_2)$ such that $O_{x(\alpha,\alpha)} \not q\, O_{y(\beta,\beta)} \Rightarrow A_1 \not q\, B_2$, since $B_1 \subseteq B_2 \Rightarrow A_1 \not q\, B_1 \Rightarrow \exists\, O_{x_\alpha} = A_1 \in \Pi_1, O_{y_\beta} = B_1 \in \Pi_1$ such that $O_{x_\alpha} \not q\, O_{y_\beta} \Rightarrow (X, \Pi_1) \in FT_2$.

The following example shows that the converse of above theorem may not be true.

Example 4.6

Let $X=\{a\},\ \tau=\{a_t : t\le 1/2\}\cup\{X\}$ be a fuzzy topological space. Then: (X,I^X) *is* FT_2. But the intuitionistic fuzzy topological space*(X,η)*, *where* $\eta = I^X \hat{\times} \tau=\{(a_r,a_t): r\le t\le 1/2\}\cup\{(a_r,X): r\in I\}$ on X is not IF-T_2 , since for $a_{(0.4,0.7)}\ \not{q}\ a_{(0.3,0.6)}$ then,

$\Rightarrow \nexists\, O_{a_{(0.4,0.7)}}$ *and* $\nexists\, O_{a_{(0.3,0.6)}}$ *such that* $O_{a_{(0.4,0.7}} \not{q}\ O_{a_{(0.3,0.6)}}$.

The following example shows that the IFTS*(X,η) is* IF-T_2 , for which *(X,Π_2) is not FT_2.*

Example 4.7

Let X be an infinite set, and $\tau_\infty=\{A\in I^X : S(A^C)\ is\ finite\}\cup\{\phi\}$ be a fuzzy topology on X, where $S(A^C)$ is the support of A^C. Then the IFTS(X, $I^X \hat{\times}\tau_\infty$) is an IF-$T_3$. But the fuzzy topological space (X, τ_∞) is not FT_2, since first $I^X \hat{\times}\tau_\infty$ *is* $IF-T_1$ *because*,

$\overline{x}_{(\alpha,\beta)}=(\overline{x}_\alpha^{\tau_\infty},\overline{x}_\alpha^{\tau_\infty}\bigcup x_\beta)=x_{(\alpha,\beta)}\ \ \forall x_{(\alpha,\beta)}$ and $I^X \hat{\times}\tau_\infty$ *is* $IF-R_2$ *because*,

for $x_{(\alpha,\beta)}\not{q}\ \underline{F}=(F_1,F_2)\in\tau_\infty^C \hat{\times} I^X \Rightarrow x_\alpha \not{q}\, F_2$ and $x_\beta\ \not{q}\, F_1 \in \tau_\infty^C$,

$\Rightarrow F_2\subseteq x_\alpha^C=O_{F_2}\in\tau_\infty$ *and* $x_\beta\subseteq F_1^C=O_{x_\beta}\in\tau_\infty.\to(1)$. Now

(X,I^X) *is* FR_2, $x_\alpha\not{q}\ F_2 \Rightarrow \exists O_{x_\alpha}=x_\alpha\in I^X$ *and* $O_{F_2}=x_\alpha^C\in\tau_\infty$ *from*(1) *with* $O_{x_\alpha}\not{q}$ O_{F_2} . Now put $O_{x_{(\alpha,\beta)}}=(O_{x_\alpha},O_{x_\beta})=(x_\alpha,F_1^C)\in I^X\hat{\times}\tau_\infty$,$O_{\underline{F}}=(O_{F_1},O_{F_2})=(F_1,x_\alpha^C)\in I^X\hat{\times}\tau_\alpha$ such that $O_{x_{(\alpha,\beta)}}\ \not{q}\ O_{\underline{F}}$.

5. The Relation between separation axioms of fuzzy topological spaces and that of induced intuitionistic fuzzy topological spaces.

Lemma 5.1

Let (X,τ) be a fuzzy topological space and ($X,\tau\hat{\times}I^X$) be the first IBTF-topological space induced by τ (see example 2.12). Then for any IFS $\underline{A}=(A_1,A_2)$ *we have:*

i) $\overline{\underline{A}}=(A_1,\overline{A}_2)$,

ii) $\underline{A}^\circ=(A_1^\circ,A_2)$, *where the closure of* A_2 *and interior of* A_1 *w .r. to* τ

Proof. Follows from theorems (2.16), (2.17).

Theorem 5.2 *Let (X,τ) be a fuzzy topological space. Then:*

(X, $\tau\hat{\times}I^X$) is $IF-R_i\Rightarrow(X,\tau)$ *is* FR_i , $i=0,1,2,3.$

Proof. As a sample we prove the cases $i = 3$ the remaining cases are similar. For $i = 3$. Let $(X, \tau \hat{\times} I^X) \in IF - R_3$, $G \, q\!\!/ \, F$ where $G, F \in \tau^C$

$\Rightarrow \underline{G} = (G, G) \, q\!\!/ \, (F, F) = \underline{F}$, $\underline{G}, \underline{F} \in I^X \hat{\times} \tau^C$. $(X, \tau \hat{\times} I^X)$ being IF-R_3, thus $\exists \, O_{\underline{G}} = (A_1, A_2)$, $O_{\underline{F}} = (B_1, B_2) \in \tau \hat{\times} I^X$ such that $O_{\underline{G}} \, q\!\!/ \, O_{\underline{F}} \Rightarrow$ $A_1 \, q\!\!/ \, B_2 \Rightarrow A_1 \, q\!\!/ \, B_1$ i.e. $\exists \, O_G = A_1 \in \tau$, $O_F = B_1 \in \tau$ such that $O_G \, q\!\!/ \, O_F$. Hence (X, τ) *is* FR_3.

The next example shows that the converse of the above theorems is not true.

Example 5.3 Let $X = \{ x, y, z \}$ and let,
$\tau = \{ X, \phi, (x_{0.5}, y_0, z_0), (x_{0.5}, y_{0.5}, z_0), (x_{0.5}, y_0, z_{0.5}), (x_{0.5}, y_{0.5}, z_{0.5}) \}$ be a fuzzy topology on X, and
$\tau^C = \{ X, \phi, (x_{0.5}, y_1, z_1), (x_{0.5}, y_{0.5}, z_1), (x_{0.5}, y_1, z_{0.5}), (x_{0.5}, y_{0.5}, z_{0.5}) \}$.
Then (X, τ) *is* FR_3. But the intuitionistic fuzzy topological space $(X, \tau \hat{\times} I^X)$ is not IF-R_0.

Theorem 5.4

Let (X, τ) be a fuzzy topological space. Then $(X, \tau \hat{\times} I^X)$ is $IF - T_0 \Rightarrow (X, \tau)$ is FT_0. Proof. Follows from theorem (4.1).

The following example shows that the converse of the above theorem may not be true.

Example 5.5

Let $X = \{ a \}$ and let $\tau = \{ a_t : t \geq 1/2 \} \cup \{ \phi \}$ be a fuzzy topology on X. Then (X, τ) *is* FT_0. But the intuitionistic fuzzy topology structure, $\tau \hat{\times} I^X = \{ (a_t, a_r) : t \leq r \geq 1/2 \} \cup \{ (\phi, a_r) : r \in I \}$ is not IF-T_0.

Theorem 5.6 *Let (X, τ) be a fuzzy topological space. Then:*
$(X, \tau \hat{\times} I^X)$ *is* $IF - T_i \Leftrightarrow (X, \tau)$ *is* FT_i , $i = 1, 2, 3, 4$.

Proof. As a sample we prove the cases i=1,2

i) For $i = 1$. Necessity follows from theorem (4.2).

Conversely, let (X, τ) *be a* FT_1, $x_{(\alpha,\beta)}$ be any IFP in $(X, \tau \hat{\times} I^X)$. Then

$\overline{x}_{(\alpha,\beta)} = (\overline{x}^2_\alpha, \overline{\overline{x}^2_\alpha \cup x_\beta}^1) = (x_\alpha, \overline{x}^1_\beta) = x_{(\alpha,\beta)}$, since (X, τ) is FT_1, hence $(X, \tau \hat{\times} I^X)$ is IF-T_1.

ii) For $i = 2$. Necessity follows from theorem (4.5).

Conversely, let (X, τ) *is a* FT_2 , $x_{(\alpha,\beta)} \, q\!\!/ \, y_{(\gamma,\lambda)}$ where ($x_{(\alpha,\beta)}$, $y_{(\gamma,\lambda)}$ are IFPS in X). Then $x_\alpha \, q\!\!/ \, y_\lambda \wedge x_\beta \, q\!\!/ \, y_\gamma \Rightarrow (\exists \, O_{x_\alpha}, O_{y_\lambda} \in \tau$ such that

$O_{x_\alpha}\, q\!\!\!/\, O_{y_\lambda}$) and ($\exists\ O_{x_\beta},\ O_{y_\gamma} \in \tau$ such that $O_{x_\beta}\, q\!\!\!/\, O_{y_\gamma}$). Now take $O^*_{x_\alpha} = O_{x_\alpha} \bigcap O_{x_\beta}$ and $O^*_{y_\gamma} = O_{y_\gamma} \bigcap O_{y_\lambda}$. Then $O_{x(\alpha,\beta)} = (O^*_{x_\alpha}, O_{x_\beta})$ and $O_{y(\gamma,\lambda)} = (O^*_{t_\gamma}, O_{y_\lambda}) \in \tau \hat{\times} I^X$ such that $O_{x(\alpha,\beta)}\, q\!\!\!/\, O_{y_{(\gamma,\lambda)}}$. The result holds.

Lemma 5.7

Let (X, τ) be a fuzzy topological space and (X, $I^X \hat{\times} \tau$) be the second IBTF-topological space induced by τ (example 2.12). Then for any IFS $\underline{A} = (A_1, A_2)$ we have:

i) $\overline{\underline{A}} = (\overline{A_1}, \overline{A_1} \bigcup A_2)$,

ii) $\underline{A}^\circ = (A_1 \bigcap A_2{}^\circ,\ A_2{}^\circ)$,

iii) $\bar{x}_{(\alpha,\alpha)} = (\bar{x}_\alpha, \bar{x}_\alpha)$ *, where the closure of* A_1, x_α *and interior of* A_2 *are with respect to* τ .

Proof. Follows from theorem (2.16), (2.17).

Theorem 5.8

Let (X, τ) be a fuzzy topological space. Then (X, $I^X \hat{\times} \tau$) $\in$ IF-R_0 $\Rightarrow$ (X, τ) $\in$ FR_0.

Proof. It is clear.

The following example shows that the converse of the above theorem may not be true.

Example 5.9 Let $X = \{ x, y, z \}$ and let,

$\tau = \{\ X, \phi, (x_{0.5}, y_0, z_0), (x_{0.5}, y_{0.5}, z_0), (x_{0.5}, y_0, z_{0.5}), (x_{0.5}, y_{0.5}, z_{0.5})\ \}$ be a fuzzy topology on X and,

$\tau^C = \{\ X, \phi, (x_{0.5}, y_1, z_1), (x_{0.5}, y_{0.5}, z_1), (x_{0.5}, y_1, z_{0.5}), (x_{0.5}, y_{0.5}, z_{0.5})\ \}$. Then (X , τ) is FR_3. But the intuitionistic fuzzy topological space (X , $I^X \hat{\times} \tau$) is not IF-R_0.

Theorem 5.10 *Let (X, τ) be a fuzzy topological space. Then:*

i) (X, $I^X \hat{\times} \tau$) is IF-T_0 $\Rightarrow$ (X, τ) is FT_0 ,

ii) (X, $I^X \hat{\times} \tau$) is IF-T_1 $\Leftrightarrow$ (X, τ) is FT_1.

Proof. It is clear..

Note: The converse of i) of the above theorem my not be true in general, this can be shown by example (4.6), where (X, τ) is FT_0 . But (X, $I^X \hat{\times} \tau$) is not IF-T_0.

Theorem 5.11

Let (X,τ) *be a fuzzy topological space. Then* (X,τ) *is* $FT_i \Rightarrow (X, I^X \hat{\times} \tau)$ *is IF-*$T_i, i = 2,3$.

Proof**.** For i=2. Let (X,τ) be a FT_2, $x_{(\alpha,\beta)}\, q\!\!\!/\, y_{(\gamma,\lambda)}$.Then $x_\alpha\, q\!\!\!/\, y_\lambda \wedge x_\beta\, q\!\!\!/\, y_\gamma$ implies $\exists\, O_{x_\alpha}, O_{y_\lambda} \in \tau$ such that $O_{x_\alpha}\, q\!\!\!/\, O_{y_\lambda}$ and $\exists\, O_{x_\beta}, O_{y_\gamma} \in \tau$ such that $O_{x_\beta}\, q\!\!\!/\, O_{y_\gamma}$. Now take, $O^*_{x_\alpha} = O_{x_\alpha} \cap O_{x_\beta}$ *and* $O^*_{y_\gamma} = O_{y_\gamma} \cap O_{y_\lambda}$, then $O_{x(\alpha,\beta)} = (O^*_{x_\alpha}, O_{x_\beta}) \in I^X \hat{\times} \tau, O_{y(\gamma,\lambda)} = (O^*_{y_\gamma}, O_{y_\lambda}) \in I^X \hat{\times} \tau$ such that $O_{x(\alpha,\beta)}\, q\!\!\!/\, O_{y_{(\gamma,\lambda)}}$, and so $(X, I^X \hat{\times} \tau)$ is IF-T_2. The rest case is similar.

Note. The example (4.7) shows that the converse of the above theorem may not be true in general.

References

[1] K. Atanassov, Intuitionistic fuzzy sets, fuzzy sets and system, Vol. 20(1986), No.1, 87-96.

[2] K. Atanassov, S. Stoeva, Intuitionistic L-fuzzy sets, in: R. Trappl. (Ed), Cybernetics and Sys. Research, Vol. 2, Elsevier, Amsterdam, (1984), 539-540.

[3] K. Atanassov, G. Gargov Interval valued intuitionistic fuzzy sets, Fuzzy sets and Systems, Vol. 31, No. 3, (1989), 343-349.

[4] S. E. Abbas, On intuitionistic fuzzy compactness, information sciences (2004), 1-16.

[5] S. Bayhan, D. Çoker, On fuzzy separation axioms in intuitionistic fuzzy topological spaces, BUSEFAL, Vol. 67, (1996), 77-87.

[6] D. Çoker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy sets and systems, Vol. 88(1997), No. 1, 81-89.

[7] D. Çoker, An introduction to intuitionistic topological spaces. BUSEFAL, Vol. 81, Hiver (2000), 51-56.

[8] D. Çoker, M. Demirci, On intuitionistic fuzzy points, Notes on IFS, Vol. 1, (1995), No.2, 79-84.

[9] D. Çoker, A note on intuitionistic sets and intuitionistic points, Turkish J. Math.Vol. 20, No. 3, (1996), 343-351.

[10] D. Çoker, A. Haydar Es, On fuzzy compactness in intuitionistic fuzzy topologicalspaces, J. of fuzzy Math. 3(4), (1995), 889-909.

[11] D. Deschrijver, E. Kerre, On relationship between some extensions of fuzzy set theory, Fuzzy sets and systems, 133(2003)227-235.

[12] J. A. Goguen, L-Fuzzy sets, J. of Math. Ana. App.18 (1967)145-174.

[13] A. Kandil, M. E. El-Shafee, Regularity axioms in fuzzy topological spaces and FR_i-Proximities, Fuzzy sets and systems, 27(1988)217-231.

[14] A. Kandil, A. M. El-Etriby, On separation axioms in fuzzy topological spaces, Tamkang J. of Math. Vol. 18(1), (1989), PP. 49-59.

[15] S. J. Lee, E. P. Lee, The category of intuitionistic fuzzy topological Bull. Korean math. Soc. 37(2000), No. 4, 63-76.

[16] S. Nanda, S. Majumdar, Fuzzy rough sets, fuzzy sets and systems, 45(1999)157-160.

[17] R. Sambuc, Function Φ-flous. Application á I' aide an diagnostic en Pathologie thyroidienne, Ph.D. Thesis, Univ. Marseille, France, 1975.

[18] L. A. Zadeh, Fuzzy sets, Inf. And control, 8(1965)338-353.