



Abhath Journal of Basic and Applied Sciences, 4(2) (2025), 40–60

Abhath Journal of Basic and Applied Sciences

Website: <https://ojs.abhath-ye.com/index.php/OJSRJBAS/about>

doi: <https://doi.org/10.59846/ajobas.v4i1.834>

Research paper



Advanced Partitioned Neutrosophic Sets: Formalization of Hexa-, Hepta-, Octa-, Nona-, and Deca-Partitioned Structures

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Received: 22 June 2025 Accepted: 06 November 2025 Published: 31 December 2025

Abstract

Uncertainty-modeling frameworks such as fuzzy sets, intuitionistic fuzzy sets, hyperfuzzy sets, neutrosophic sets, and plithogenic sets have been widely adopted for representing and reasoning under vagueness and imprecision. Neutrosophic sets assign each element three independent membership values—truth, indeterminacy, and falsity—and their partitioned extensions introduce additional components, subject to sum-bounded constraints. Notable examples include quadripartitioned, pentapartitioned, and heptapartitioned neutrosophic sets. In this work, we formally define the hexa-, octa-, nona-, and decapartitioned neutrosophic sets and prove that each can be naturally embedded within the plithogenic set framework. This unifying representation facilitates a comprehensive approach to neutrosophic set theory and broadens its potential applications.

Keywords: Neutrosophic Set, Quadripartitioned Neutrosophic Set, Pentapartitioned Neutrosophic Set, Heptapartitioned Neutrosophic set

1 Introduction

1.1 Neutrosophic Set Theory

Uncertainty arises in many real-world problems, prompting the creation of diverse mathematical frameworks to handle imprecision. Classical approaches such as *fuzzy sets* [1, 2, 3] and *intuitionistic fuzzy sets* [4, 5, 6] laid the foundation, while more recent variants—*vague sets* [7], *bipolar fuzzy sets* [8], *hesitant fuzzy sets* [9, 10], *spherical fuzzy sets* [11, 12], *picture fuzzy sets* [13, 14], and *m-polar fuzzy sets* [15, 16, 17]—introduce additional layers of nuance. Extensions into the neutrosophic realm yield *neutrosophic sets* [18, 19, 20], which assign to each element three independent measures—truth, indeterminacy, and falsity—each valued in $[0, 1]$, with their sum bounded by 3. Further refinements such as *hyperneutrosophic sets* [21, 22, 23] have broadened the expressive power of these models.

Beyond these, *rough sets* [24, 25] and their variants—*hyperrough sets* [26]—address vagueness through lower and upper approximations, while *soft sets* [27, 28] and *hypersoft sets* [29, 30] leverage parameterization

to capture context-dependent uncertainty. Recently, *plithogenic sets* [31, 32, 33] have unified many of these ideas by incorporating a contradiction function alongside multi-attribute membership degrees.

To distinguish subtle facets of ignorance and conflict, partitioned neutrosophic frameworks have been developed:

- **Quadripartitioned sets** introduce a *contradiction* component, limiting the sum of four degrees to 4 [34, 35].
- **Pentapartitioned sets** add an *unknown* measure, with five components summing to at most 5 [36, 37].
- **Heptapartitioned sets** further distinguish *relative truth* and *relative falsity*, yielding seven degrees bounded by 7 [38].

Moreover, the notions of *double-valued neutrosophic sets* [39, 40, 41, 42, 43, 44] and *triple-valued neutrosophic sets* [45, 46, 47, 48] have also been introduced and studied.

1.2 Our Contribution

This paper proposes four novel partitioned structures—*hexapartitioned*, *octapartitioned*, *nonapartitioned*, and *decapartitioned* neutrosophic sets—and demonstrates how each can be embedded within the plithogenic set framework. By presenting a unified construction, we establish a consistent foundation for these extended models and pave the way for their application in complex-system analysis and decision support.

2 Preliminaries

We begin by recalling the definitions of the already established concepts of Neutrosophic Sets[49], Quadripartitioned Neutrosophic Sets[50, 51, 52, 53], Pentapartitioned Neutrosophic Sets [54, 55, 36], and Heptapartitioned Neutrosophic Sets [56, 57]. Throughout this paper, all sets are assumed to be finite. For the basic operations associated with each concept, the reader is referred to the respective references.

Definition 2.1 (Neutrosophic Set). [58, 18] *Let X be a non-empty set. A Neutrosophic Set (NS) A on X is characterized by three membership functions:*

$$T_A : X \rightarrow [0, 1], \quad I_A : X \rightarrow [0, 1], \quad F_A : X \rightarrow [0, 1],$$

where for each $x \in X$, the values $T_A(x)$, $I_A(x)$, and $F_A(x)$ represent the degrees of truth, indeterminacy, and falsity, respectively. These values satisfy the following condition:

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$

Definition 2.2 (Quadripartitioned Neutrosophic Set). [59, 60, 34, 61] *Let U be a non-empty universe. A quadripartitioned neutrosophic set (Q -NS) Q on U is defined by*

$$Q = \{ \langle x, T_Q(x), C_Q(x), G_Q(x), F_Q(x) \rangle \mid x \in U \},$$

where

$$T_Q, C_Q, G_Q, F_Q : U \longrightarrow [0, 1]$$

are the truth, contradiction, ignorance, and falsity membership functions, respectively, satisfying

$$0 \leq T_Q(x) + C_Q(x) + G_Q(x) + F_Q(x) \leq 4 \quad \text{for all } x \in U.$$

Definition 2.3 (Pentapartitioned Neutrosophic Set). [62, 54] Let U be a non-empty universe. A pentapartitioned neutrosophic set (P -NS) P on U is defined by

$$P = \{ \langle x, T_P(x), C_P(x), G_P(x), U_P(x), F_P(x) \rangle \mid x \in U \},$$

where

$$T_P, C_P, G_P, U_P, F_P : U \longrightarrow [0, 1]$$

are the truth, contradiction, ignorance, unknown, and falsity membership functions, respectively, satisfying

$$0 \leq T_P(x) + C_P(x) + G_P(x) + U_P(x) + F_P(x) \leq 5 \quad \text{for all } x \in U.$$

Definition 2.4 (Heptapartitioned Neutrosophic Set). [63, 64, 65] Let U be a non-empty universe. A heptapartitioned neutrosophic set (H -NS) H on U is defined by

$$H = \{ \langle x, T_H(x), M_H(x), C_H(x), U_H(x), I_H(x), K_H(x), F_H(x) \rangle \mid x \in U \},$$

where

$$T_H, M_H, C_H, U_H, I_H, K_H, F_H : U \longrightarrow [0, 1]$$

are the truth, relative truth, contradiction, unknown, ignorance, relative falsity, and falsity membership functions, respectively, satisfying

$$0 \leq T_H(x) + M_H(x) + C_H(x) + U_H(x) + I_H(x) + K_H(x) + F_H(x) \leq 7 \quad \text{for all } x \in U.$$

Example 2.5 (Heptapartitioned Neutrosophic Set in Candidate Evaluation). *Candidate Evaluation refers to the process of assessing individuals' qualifications, skills, and suitability for specific roles or opportunities (cf.[66, 67, 68]). Consider three job candidates:*

$$U = \{\text{Alice}, \text{Bob}, \text{Carol}\}.$$

We model the hiring committee's uncertainty with a heptapartitioned neutrosophic set

$$H = \{ \langle x, T_H(x), M_H(x), C_H(x), U_H(x), I_H(x), K_H(x), F_H(x) \rangle \mid x \in U \},$$

where each membership function represents:

- $T_H(x)$: strong positive recommendation (truth),
- $M_H(x)$: moderate positive evaluation (relative truth),
- $C_H(x)$: conflicting feedback (contradiction),
- $U_H(x)$: aspects not reviewed (unknown),
- $I_H(x)$: lack of information (ignorance),
- $K_H(x)$: moderate negative evaluation (relative falsity),
- $F_H(x)$: strong negative recommendation (falsity).

A possible assignment is:

$$\begin{aligned} H(\text{Alice}) &= (0.6, 0.2, 0.1, 0.0, 0.1, 0.0, 0.0), \\ H(\text{Bob}) &= (0.3, 0.0, 0.2, 0.1, 0.1, 0.2, 0.1), \\ H(\text{Carol}) &= (0.0, 0.1, 0.3, 0.2, 0.2, 0.1, 0.1). \end{aligned}$$

Here, for each candidate x ,

$$0 \leq T_H(x) + M_H(x) + C_H(x) + U_H(x) + I_H(x) + K_H(x) + F_H(x) \leq 7.$$

Thus H captures varied degrees of support, conflict, and gaps in information for each candidate.

3 Main Results of this Paper

The results of this paper are presented as follows.

3.1 Hexapartitioned Neutrosophic Set

A Hexapartitioned Neutrosophic Set assigns six membership degrees—truth, contradiction, ignorance, unknown, hesitation, and falsity—to each element, such that their sum does not exceed six. This concept is developed with reference to works such as [69]. The formal definition is provided below.

Definition 3.1 (Hexapartitioned Neutrosophic Set). *(cf.[69]) Let U be a non-empty universe. A hexapartitioned neutrosophic set N on U is an object of the form*

$$N = \{ \langle x, T(x), C(x), G(x), U(x), H(x), F(x) \rangle \mid x \in U \},$$

where

$$T, C, G, U, H, F : U \longrightarrow [0, 1]$$

are the truth, contradiction, ignorance, unknown, hesitation, and falsity membership functions, respectively, subject to

$$0 \leq T(x) + C(x) + G(x) + U(x) + H(x) + F(x) \leq 6 \quad \text{for all } x \in U.$$

Here we have replaced the “ambiguous” component found in some hexa-valued schemes by the ignorance membership $G(x)$.

Example 3.2 (Medical Diagnosis for Disease X). *Medical diagnosis is the process of identifying a disease or condition by evaluating a patient’s symptoms, history, and test results (cf.[70, 71, 72]). Consider three patients undergoing evaluation for Disease X:*

$$U = \{\text{Alice}, \text{Bob}, \text{Charlie}\}.$$

We define a hexapartitioned neutrosophic set

$$N = \{ \langle x, T(x), C(x), G(x), U(x), H(x), F(x) \rangle \mid x \in U \},$$

where the membership functions are interpreted as:

- $T(x)$: degree of positive test evidence (truth),
- $C(x)$: degree of conflicting test results (contradiction),
- $G(x)$: degree of missing test information (ignorance),
- $U(x)$: degree of unperformed tests (unknown),
- $H(x)$: degree of inconclusive or borderline results (hesitation),
- $F(x)$: degree of negative test evidence (falsity).

A concrete assignment might be:

$$\begin{aligned} N = \{ & \langle \text{Alice}, 0.9, 0.0, 0.0, 0.1, 0.0, 0.0 \rangle, \\ & \langle \text{Bob}, 0.5, 0.3, 0.0, 0.0, 0.2, 0.0 \rangle, \\ & \langle \text{Charlie}, 0.0, 0.0, 0.8, 0.1, 0.1, 0.0 \rangle \}, \end{aligned}$$

where for each patient x :

$$T(x) + C(x) + G(x) + U(x) + H(x) + F(x) \leq 6.$$

- Alice: high positive evidence ($T = 0.9$), minor unknowns ($U = 0.1$), no conflict or hesitation.
- Bob: moderate positive ($T = 0.5$), conflicting results ($C = 0.3$), some hesitation ($H = 0.2$).
- Charlie: no test performed ($U = 0.1$), large missing information ($G = 0.8$), slight hesitation ($H = 0.1$).

This set N captures, in a unified way, varying degrees of test outcomes, missing data, and inconclusiveness in the medical diagnosis context.

Theorem 3.3 (Hexapartitioned Generalizes Pentapartitioned). *Let U be a non-empty universe and let*

$$P = \{ \langle x, T_P(x), C_P(x), G_P(x), U_P(x), F_P(x) \rangle \mid x \in U \}$$

be a pentapartitioned neutrosophic set on U , with

$$T_P, C_P, G_P, U_P, F_P : U \rightarrow [0, 1], \quad T_P(x) + C_P(x) + G_P(x) + U_P(x) + F_P(x) \leq 5 \quad \forall x \in U.$$

Define functions

$$T(x) := T_P(x), \quad C(x) := C_P(x), \quad G(x) := G_P(x), \quad U(x) := U_P(x), \quad H(x) := 0, \quad F(x) := F_P(x),$$

and let

$$N = \{ \langle x, T(x), C(x), G(x), U(x), H(x), F(x) \rangle \mid x \in U \}.$$

Then N is a hexapartitioned neutrosophic set on U which restricts to P when the sixth component H is omitted.

Proof. We must show (i) N satisfies the hexapartitioned membership sum bound and (ii) it recovers P by projection.

(i) Validity as a hexapartitioned set. By construction,

$$T(x), C(x), G(x), U(x), H(x), F(x) \in [0, 1],$$

and for each x ,

$$T(x) + C(x) + G(x) + U(x) + H(x) + F(x) = T_P(x) + C_P(x) + G_P(x) + U_P(x) + 0 + F_P(x) \leq 5 \leq 6.$$

Therefore N meets the requirement $\sum_{\ell \in \{T, C, G, U, H, F\}} \ell(x) \leq 6$, so N is a valid hexapartitioned neutrosophic set.

(ii) Projection recovers P . Consider the projection map $\pi : [T, C, G, U, H, F] \mapsto [T, C, G, U, F]$. By definition of N , for every x ,

$$\pi(T(x), C(x), G(x), U(x), H(x), F(x))) = (T_P(x), C_P(x), G_P(x), U_P(x), F_P(x)).$$

Hence the pentatuple from P is exactly obtained from the sextuple in N . This shows P is embedded in N .

Since both (i) and (ii) hold, hexapartitioned sets indeed generalize pentapartitioned sets. $\square$

3.2 Octapartitioned Neutrosophic Set

An Octapartitioned Neutrosophic Set assigns eight membership degrees—truth, relative truth, contradiction, unknown, ignorance, hesitancy, relative falsity, falsity—to each element.

Definition 3.4 (Octapartitioned Neutrosophic Set). *Let U be a non-empty universe. An octapartitioned neutrosophic set (O-NS) O on U is defined by*

$$O = \{ \langle x, T_O(x), M_O(x), C_O(x), U_O(x), I_O(x), K_O(x), H_O(x), F_O(x) \rangle \mid x \in U \},$$

where

$$T_O, M_O, C_O, U_O, I_O, K_O, H_O, F_O : U \longrightarrow [0, 1]$$

are the truth, relative truth, contradiction, unknown, ignorance, relative falsity, hesitancy, and falsity membership functions, respectively, satisfying for all $x \in U$:

$$0 \leq T_O(x) + M_O(x) + C_O(x) + U_O(x) + I_O(x) + K_O(x) + H_O(x) + F_O(x) \leq 8.$$

Here:

- $T_O(x)$ is the degree of truth,
- $M_O(x)$ is the degree of relative truth,
- $C_O(x)$ is the degree of contradiction,
- $U_O(x)$ is the degree of unknown,
- $I_O(x)$ is the degree of ignorance,
- $H_O(x)$ is the degree of hesitancy,
- $K_O(x)$ is the degree of relative falsity,
- $F_O(x)$ is the degree of falsity.

Example 3.5 (Loan Application Risk Assessment). *Risk assessment is the systematic process of identifying, analyzing, and evaluating potential hazards to minimize impact and ensure informed decisions (cf.[73, 74, 75]). A bank evaluates three loan applicants. Let*

$$U = \{\text{Alice, Bob, Charlie}\}$$

be the set of applicants. We model the assessment as an octapartitioned neutrosophic set

$$O = \{ \langle x, T_O(x), M_O(x), C_O(x), U_O(x), I_O(x), K_O(x), H_O(x), F_O(x) \rangle \mid x \in U \},$$

where each membership function represents:

- $T_O(x)$: confidence in approving the loan (truth),
- $M_O(x)$: relative confidence (strong applicant) (relative truth),
- $C_O(x)$: presence of conflicting information (contradiction),
- $U_O(x)$: missing documentation (unknown),

- $I_O(x)$: insufficient credit history (ignorance),
- $K_O(x)$: relative risk factors (relative falsity),
- $H_O(x)$: borderline credit score (hesitancy),
- $F_O(x)$: reasons for outright rejection (falsity).

A specific assignment might be:

$$O = \{ \langle \text{Alice}, 0.7, 0.2, 0.0, 0.0, 0.1, 0.0, 0.0, 0.0 \rangle, \\ \langle \text{Bob}, 0.4, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.0 \rangle, \\ \langle \text{Charlie}, 0.1, 0.0, 0.2, 0.3, 0.2, 0.1, 0.0, 0.1 \rangle \}.$$

Here:

- Alice has high approval confidence ($T_O = 0.7$) and strong credit history ($M_O = 0.2$), with a small gap in credit history ($I_O = 0.1$).
- Bob shows mixed signals: moderate approval ($T_O = 0.4$), some conflicting data ($C_O = 0.1$), missing document ($U_O = 0.1$), and borderline factors ($H_O = 0.1, K_O = 0.1$).
- Charlie has low approval ($T_O = 0.1$), notable contradictions in application ($C_O = 0.2$), substantial unknowns ($U_O = 0.3$), and some outright rejection reasons ($F_O = 0.1$).

Each tuple's components lie in $[0, 1]$ and their sum does not exceed 8, making O a valid octapartitioned neutrosophic set that captures the nuanced, multi-faceted uncertainty in loan risk assessment.

Theorem 3.6 (Octapartitioned Generalizes Hexapartitioned). *Let U be a non-empty universe and let*

$$H = \{ \langle x, T_H(x), C_H(x), G_H(x), U_H(x), H_H(x), F_H(x) \rangle \mid x \in U \}$$

be a hexapartitioned neutrosophic set on U , where

$$T_H, C_H, G_H, U_H, H_H, F_H : U \longrightarrow [0, 1]$$

are the truth, contradiction, ignorance, unknown, hesitation, and falsity membership functions, respectively, satisfying

$$T_H(x) + C_H(x) + G_H(x) + U_H(x) + H_H(x) + F_H(x) \leq 6 \quad \forall x \in U.$$

Define on U the eight functions

$$\begin{aligned} T_O(x) &:= T_H(x), & M_O(x) &:= 0, \\ C_O(x) &:= C_H(x), & U_O(x) &:= U_H(x), \\ I_O(x) &:= G_H(x), & K_O(x) &:= 0, \\ H_O(x) &:= H_H(x), & F_O(x) &:= F_H(x). \end{aligned}$$

Then

$$O = \{ \langle x, T_O(x), M_O(x), C_O(x), U_O(x), I_O(x), K_O(x), H_O(x), F_O(x) \rangle \mid x \in U \}$$

is an octapartitioned neutrosophic set on U which restricts to H when the two added components M_O and K_O are omitted.

Proof. We verify that O satisfies the octapartitioned requirements and projects back to H .

(i) Validity as an octapartitioned set. By construction each $T_O, C_O, \dots, F_O$ takes values in $[0, 1]$. Moreover, for every $x \in U$:

$$\begin{aligned} T_O(x) + M_O(x) + C_O(x) + U_O(x) + \\ I_O(x) + K_O(x) + H_O(x) + F_O(x) &= T_H(x) + C_H(x) + G_H(x) \\ &\quad + U_H(x) + H_H(x) + F_H(x) \\ &\leq 6 \leq 8. \end{aligned}$$

Hence O satisfies the sum-bound ≤ 8 of an octapartitioned neutrosophic set.

(ii) Projection recovers H . Define the projection $\pi : [T, M, C, U, I, K, H, F] \mapsto [T, C, G, U, H, F]$. Then for each $x \in U$:

$$\begin{aligned} \pi(T_O(x), M_O(x), C_O(x), U_O(x), I_O(x), K_O(x), H_O(x), F_O(x)) \\ = (T_H(x), C_H(x), G_H(x), U_H(x), H_H(x), F_H(x)), \end{aligned}$$

which exactly reproduces the six-tuple defining H .

Since both the octapartitioned conditions and the recovery of H hold, we conclude that every hexapartitioned neutrosophic set can be seen as a special case of an octapartitioned neutrosophic set. $\square$

Theorem 3.7 (Octapartitioned Generalizes Heptapartitioned). *Let U be a non-empty universe and let*

$$H = \{ \langle x, T_H(x), M_H(x), C_H(x), U_H(x), I_H(x), K_H(x), F_H(x) \rangle \mid x \in U \}$$

be a heptapartitioned neutrosophic set on U , where

$$T_H, M_H, C_H, U_H, I_H, K_H, F_H : U \rightarrow [0, 1]$$

and

$$T_H(x) + M_H(x) + C_H(x) + U_H(x) + I_H(x) + K_H(x) + F_H(x) \leq 7 \quad \forall x \in U.$$

Define new functions on U by

$$\begin{aligned} T_O(x) &:= T_H(x), & M_O(x) &:= M_H(x), & C_O(x) &:= C_H(x), & U_O(x) &:= U_H(x), \\ I_O(x) &:= I_H(x), & K_O(x) &:= K_H(x), & H_O(x) &:= 0, & F_O(x) &:= F_H(x). \end{aligned}$$

Then

$$O = \{ \langle x, T_O(x), M_O(x), C_O(x), U_O(x), I_O(x), K_O(x), H_O(x), F_O(x) \rangle \mid x \in U \}$$

is an octapartitioned neutrosophic set on U which restricts to H when the hesitancy component H_O is omitted.

Proof. We verify two requirements:

(i) Validity as an octapartitioned set. By construction, for each $x \in U$,

$$T_O(x), M_O(x), C_O(x), U_O(x), I_O(x), K_O(x), H_O(x), F_O(x) \in [0, 1],$$

and

$$\begin{aligned} T_O(x) + M_O(x) + C_O(x) + U_O(x) + I_O(x) + K_O(x) + H_O(x) + F_O(x) \\ = T_H(x) + M_H(x) + C_H(x) + U_H(x) + I_H(x) + K_H(x) + 0 + F_H(x) \\ \leq 7 \leq 8. \end{aligned}$$

Thus O satisfies the octapartitioned sum-bound ≤ 8 .

(ii) Projection recovers H . Define the projection $\pi : [T, M, C, U, I, K, H, F] \mapsto [T, M, C, U, I, K, F]$. Then for each x ,

$$\begin{aligned} \pi(T_O(x), M_O(x), C_O(x), U_O(x), I_O(x), K_O(x), H_O(x), F_O(x)) \\ = (T_H(x), M_H(x), C_H(x), U_H(x), I_H(x), K_H(x), F_H(x)), \end{aligned}$$

which are exactly the seven components of H . Hence H embeds into O .

Having established both (i) and (ii), we conclude that every heptapartitioned neutrosophic set can be viewed as a special case of an octapartitioned neutrosophic set. $\square$

3.3 Nonapartitioned Neutrosophic Set

A Nonapartitioned Neutrosophic Set assigns nine membership degrees—truth, two truth levels, contradiction, unknown, ignorance, two falsity levels, and falsity—to each element.

Definition 3.8 (Nonapartitioned Neutrosophic Set). *Let U be a non-empty universe. A nonapartitioned neutrosophic set (N -NS) N on U is defined by*

$$N = \{ \langle x, T_N(x), ST_N(x), WT_N(x), C_N(x), U_N(x), I_N(x), SF_N(x), WF_N(x), F_N(x) \rangle \mid x \in U \},$$

where

$$T_N, ST_N, WT_N, C_N, U_N, I_N, SF_N, WF_N, F_N : U \longrightarrow [0, 1]$$

are the truth, strongly relative truth, weakly relative truth, contradiction, unknown, ignorance, strongly relative falsity, weakly relative falsity, and falsity membership functions, respectively, subject to

$$0 \leq T_N(x) + ST_N(x) + WT_N(x) + C_N(x) + U_N(x) + I_N(x) + SF_N(x) + WF_N(x) + F_N(x) \leq 9 \quad \text{for all } x \in U.$$

Example 3.9 (Nonapartitioned Neutrosophic Set: Product Review). *Customer evaluation is the process of assessing customer satisfaction, feedback, and behavior to improve products, services, and overall business performance (cf.[76, 77, 78]). Let*

$$U = \{ \text{Product}_A, \text{Product}_B, \text{Product}_C \}$$

be three products under customer evaluation. We define a nonapartitioned neutrosophic set

$$N = \{ \langle x, T_N(x), ST_N(x), WT_N(x), C_N(x), U_N(x), I_N(x), SF_N(x), WF_N(x), F_N(x) \rangle \mid x \in U \},$$

where each membership degree represents:

- $T_N(x)$: strong positive reviews (truth),
- $ST_N(x)$: very high satisfaction (strongly relative truth),
- $WT_N(x)$: mild satisfaction (weakly relative truth),
- $C_N(x)$: conflicting opinions (contradiction),
- $U_N(x)$: missing ratings (unknown),
- $I_N(x)$: lack of expert analysis (ignorance),
- $SF_N(x)$: severe flaws noted (strongly relative falsity),

- $WF_N(x)$: minor issues noted (weakly relative falsity),
- $F_N(x)$: outright negative consensus (falsity).

A concrete instantiation is:

$$N = \{ \langle \text{Product}_A, 0.5, 0.3, 0.1, 0.0, 0.0, 0.0, 0.0, 0.0 \rangle, \\ \langle \text{Product}_B, 0.2, 0.1, 0.1, 0.2, 0.1, 0.1, 0.1, 0.1 \rangle, \\ \langle \text{Product}_C, 0.0, 0.0, 0.2, 0.0, 0.2, 0.3, 0.1, 0.1 \rangle \},$$

where for each $x \in U$,

$$0 \leq T_N(x) + ST_N(x) + WT_N(x) + C_N(x) + U_N(x) + I_N(x) + SF_N(x) + WF_N(x) + F_N(x) \leq 9.$$

This example captures nuanced customer sentiments including missing data and conflicting feedback.

Theorem 3.10 (Nonapartitioned Generalizes Heptapartitioned). *Let U be a non-empty universe and let*

$$H = \{ \langle x, T_H(x), M_H(x), C_H(x), U_H(x), I_H(x), K_H(x), F_H(x) \rangle \mid x \in U \}$$

be a heptapartitioned neutrosophic set on U , with

$$T_H, M_H, C_H, U_H, I_H, K_H, F_H : U \rightarrow [0, 1], \quad T_H(x) + M_H(x) + C_H(x) + U_H(x) + I_H(x) + K_H(x) + F_H(x) \leq 7.$$

Define for each $x \in U$:

$$\begin{aligned} T_N(x) &:= T_H(x), & ST_N(x) &:= M_H(x), & WT_N(x) &:= 0, \\ C_N(x) &:= C_H(x), & U_N(x) &:= U_H(x), & I_N(x) &:= I_H(x), \\ SF_N(x) &:= K_H(x), & WF_N(x) &:= 0, & F_N(x) &:= F_H(x). \end{aligned}$$

Then

$$N = \{ \langle x, T_N(x), ST_N(x), WT_N(x), C_N(x), U_N(x), I_N(x), SF_N(x), WF_N(x), F_N(x) \rangle \mid x \in U \}$$

is a nonapartitioned neutrosophic set on U which projects to H by omitting the zero-valued components.

Proof. By construction, each $T_N, ST_N, \dots, F_N$ maps into $[0, 1]$. Moreover, for every x :

$$\begin{aligned} T_N(x) + ST_N(x) + WT_N(x) + C_N(x) + U_N(x) + I_N(x) + SF_N(x) + WF_N(x) + F_N(x) \\ = T_H(x) + M_H(x) + 0 + C_H(x) + U_H(x) + I_H(x) + K_H(x) + 0 + F_H(x) \\ = T_H(x) + M_H(x) + C_H(x) + U_H(x) + I_H(x) + K_H(x) + F_H(x) \leq 7 \leq 9. \end{aligned}$$

Hence N satisfies the nonapartitioned sum-bound. The projection that deletes WT_N and WF_N recovers exactly the seven-tuple defining H . Therefore every heptapartitioned neutrosophic set embeds into a nonapartitioned neutrosophic set. $\square$

3.4 Decapartitioned Neutrosophic Set

A Decapartitioned Neutrosophic Set assigns ten membership degrees—truth, two levels of truth, contradiction, unknown, ignorance, hesitation, two levels of falsity, and falsity—to each element.

Definition 3.11 (Decapartitioned Neutrosophic Set). *Let U be a non-empty universe. A decapartitioned neutrosophic set (D -NS) D on U is defined by*

$$D = \{ \langle x, T(x), \text{SRT}(x), \text{WRT}(x), C(x), U(x), I(x), H(x), \text{SRF}(x), \text{WRF}(x), F(x) \rangle \mid x \in U \},$$

where the ten membership functions

$$T, \text{SRT}, \text{WRT}, C, U, I, H, \text{SRF}, \text{WRF}, F : U \longrightarrow [0, 1]$$

represent:

- $T(x)$: truth,
- $\text{SRT}(x)$: strongly relative truth,
- $\text{WRT}(x)$: weakly relative truth,
- $C(x)$: contradiction,
- $U(x)$: unknown,
- $I(x)$: ignorance,
- $H(x)$: hesitancy,
- $\text{SRF}(x)$: strongly relative falsity,
- $\text{WRF}(x)$: weakly relative falsity,
- $F(x)$: falsity,

subject to the condition

$$0 \leq T(x) + \text{SRT}(x) + \text{WRT}(x) + C(x) + U(x) + I(x) + H(x) + \text{SRF}(x) + \text{WRF}(x) + F(x) \leq 10,$$

for all $x \in U$.

Example 3.12 (Smartphone Review Evaluation). (cf.[79, 80]) *A retailer evaluates three smartphone models based on customer and expert feedback. Let*

$$U = \{ \text{Phone}_A, \text{Phone}_B, \text{Phone}_C \}.$$

We represent the overall evaluation as a decapartitioned neutrosophic set

$$D = \{ \langle x, T(x), \text{SRT}(x), \text{WRT}(x), C(x), U(x), I(x), H(x), \text{SRF}(x), \text{WRF}(x), F(x) \rangle \mid x \in U \},$$

with membership functions interpreted as follows:

- $T(x)$: strong positive consensus (truth),
- $\text{SRT}(x)$: strongly relative positive (exceeds expectations),
- $\text{WRT}(x)$: weakly relative positive (slightly above average),
- $C(x)$: contradictory opinions,
- $U(x)$: unknown (no data for some features),

- $I(x)$: ignorance (lack of expert reviews),
- $H(x)$: hesitancy (mixed but not contradictory),
- $\text{SRF}(x)$: strongly relative negative (major flaws),
- $\text{WRF}(x)$: weakly relative negative (minor issues),
- $F(x)$: outright negative consensus (falsity).

A concrete assignment might be:

$$D = \{ \langle \text{Phone}_A, 0.4, 0.2, 0.1, 0.0, 0.0, 0.0, 0.1, 0.0, 0.0, 0.0 \rangle, \\ \langle \text{Phone}_B, 0.1, 0.0, 0.2, 0.1, 0.2, 0.1, 0.1, 0.1, 0.1, 0.0 \rangle, \\ \langle \text{Phone}_C, 0.0, 0.0, 0.0, 0.2, 0.3, 0.2, 0.1, 0.1, 0.1, 0.1 \rangle \},$$

where for each $x \in U$:

$$0 \leq T(x) + \text{SRT}(x) + \text{WRT}(x) + C(x) + U(x) + I(x) + H(x) + \text{SRF}(x) + \text{WRF}(x) + F(x) \leq 10.$$

- Phone_A enjoys strong positive consensus ($T = 0.4$, $\text{SRT} = 0.2$) and minor hesitancy ($H = 0.1$).
- Phone_B shows mixed feedback: moderate positive ($\text{WRT} = 0.2$), some unknowns ($U = 0.2$), minor negatives ($\text{SRF} = 0.1$, $\text{WRF} = 0.1$), and hesitancy ($H = 0.1$).
- Phone_C suffers significant unknowns ($U = 0.3$), ignorance ($I = 0.2$), contradiction ($C = 0.2$), and outright negative consensus ($F = 0.1$), with slight hesitancy ($H = 0.1$).

This decapartitioned neutrosophic set thus captures ten nuanced facets of uncertainty and sentiment in smartphone reviews.

Theorem 3.13 (Decapartitioned Generalizes Non- and Octa-partitioned). *Let U be a non-empty universe. Then*

1. every nonapartitioned neutrosophic set on U , and
2. every octapartitioned neutrosophic set on U ,

embeds naturally into a decapartitioned neutrosophic set on U .

Proof. We treat the two cases separately by constructing zero-extensions of the missing components.

(i) Embedding a Nonapartitioned Set. Let

$$N = \{ \langle x, T_N(x), \text{SRT}_N(x), \text{WRT}_N(x), C_N(x), U_N(x), I_N(x), H_N(x), \text{SRF}_N(x), \text{WRF}_N(x), F_N(x) \rangle \}$$

be a nonapartitioned neutrosophic set on U , where the nine functions satisfy

$$\sum_{f \in \{T_N, \text{SRT}_N, \text{WRT}_N, C_N, U_N, I_N, H_N, \text{SRF}_N, \text{WRF}_N, F_N\}} f(x) \leq 9$$

for all $x \in U$. Define

$$T_D(x) := T_N(x), \quad \text{SRT}_D(x) := \text{SRT}_N(x), \quad \text{WRT}_D(x) := \text{WRT}_N(x), \\ C_D(x) := C_N(x), \quad U_D(x) := U_N(x), \quad I_D(x) := I_N(x), \quad H_D(x) := H_N(x),$$

$$\text{SRF}_D(x) := \text{SRF}_N(x), \quad \text{WRF}_D(x) := \text{WRF}_N(x), \quad F_D(x) := F_N(x).$$

Then

$$\sum_{g \in \{T_D, \text{SRT}_D, \text{WRT}_D, C_D, U_D, I_D, H_D, \text{SRF}_D, \text{WRF}_D, F_D\}} g(x) = \sum_{f \in N\text{-components}} f(x) \leq 9 \leq 10,$$

so the resulting decapartitioned set $\{\langle x, T_D, \dots, F_D \rangle\}$ is valid.

(ii) Embedding an Octapartitioned Set. Let

$$O = \{ \langle x, T_O(x), M_O(x), C_O(x), U_O(x), I_O(x), K_O(x), H_O(x), F_O(x) \rangle \}$$

be an octapartitioned neutrosophic set, satisfying $\sum T_O + M_O + C_O + U_O + I_O + K_O + H_O + F_O \leq 8$. Define

$$\begin{aligned} T_D(x) &:= T_O(x), & \text{SRT}_D(x) &:= M_O(x), & \text{WRT}_D(x) &:= 0, & C_D(x) &:= C_O(x), \\ U_D(x) &:= U_O(x), & I_D(x) &:= I_O(x), & H_D(x) &:= H_O(x), & \text{SRF}_D(x) &:= K_O(x), \\ & & \text{WRF}_D(x) &:= 0, & F_D(x) &:= F_O(x). \end{aligned}$$

Then

$$\sum_{\text{all } 10 \text{ } g} g(x) = T_O + M_O + 0 + C_O + U_O + I_O + H_O + K_O + 0 + F_O \leq 8 \leq 10,$$

so the extension is a valid decapartitioned neutrosophic set.

In both cases, projecting away the zero-components recovers the original nonapartitioned or octapartitioned set. Hence decapartitioned sets generalize both. $\square$

4 Additional Result: Representation of All Partitioned Neutrosophic Sets as Plithogenic Sets

A plithogenic set assigns to each element degrees of multiple attributes and a contradiction measure, enabling nuanced robust uncertainty representation [81]. This framework realizes a representation of all partitioned neutrosophic sets as plithogenic sets.

Definition 4.1 (Plithogenic Set). [82, 32] Let S be a universal set, and $P \subseteq S$. A Plithogenic Set PS is defined as:

$$PS = (P, v, P_v, pdf, pCF)$$

where:

- v is an attribute.
- P_v is the range of possible values for the attribute v .
- $pdf : P \times P_v \rightarrow [0, 1]^s$ is the Degree of Appurtenance Function (DAF).
- $pCF : P_v \times P_v \rightarrow [0, 1]^t$ is the Degree of Contradiction Function (DCF).

These functions satisfy the following axioms for all $a, b \in P_v$:

1. Reflexivity of Contradiction Function:

$$pCF(a, a) = 0$$

2. Symmetry of Contradiction Function:

$$pCF(a, b) = pCF(b, a)$$

Example 4.2 (Plithogenic Set in Employee Skill Assessment). *Employee Skill Assessment is the process of evaluating workers' abilities, knowledge, and competencies to match roles and improve performance (cf.[83, 84, 85]). Let S be the set of all employees and consider the subset*

$$P = \{Alice, Bob, Carol\}.$$

Choose the attribute

$$v = \text{"Primary Programming Language"},$$

with range of values

$$P_v = \{Java, Python, SQL\}.$$

We take $s = 1$ and $t = 1$, so that

$$pdf : P \times P_v \rightarrow [0, 1], \quad pCF : P_v \times P_v \rightarrow [0, 1].$$

Define the Degree of Appurtenance Function by

$$\begin{aligned} pdf(Alice, Java) &= 0.8, & pdf(Alice, Python) &= 0.6, & pdf(Alice, SQL) &= 0.4, \\ pdf(Bob, Java) &= 0.5, & pdf(Bob, Python) &= 0.7, & pdf(Bob, SQL) &= 0.3, \\ pdf(Carol, Java) &= 0.2, & pdf(Carol, Python) &= 0.4, & pdf(Carol, SQL) &= 0.9. \end{aligned}$$

Define the Degree of Contradiction Function by

$$\begin{aligned} pCF(Java, Python) &= 0.2, & pCF(Java, SQL) &= 0.6, \\ pCF(Python, SQL) &= 0.4, \end{aligned}$$

and $pCF(a, a) = 0$, symmetry holds. Then

$$PS = (P, v, P_v, pdf, pCF)$$

is a valid Plithogenic Set capturing each employee's primary language proficiency along with inter-language contradiction degrees.

Theorem 4.3. *Let U be a nonempty universe. For each integer k with $4 \leq k \leq 10$, every k -partitioned neutrosophic set on U can be embedded into a plithogenic set.*

Proof. Fix $k \in \{4, 5, \dots, 10\}$. Let

$$N_k = \{ \langle x, M_1(x), M_2(x), \dots, M_k(x) \rangle \mid x \in U \}$$

be any k -partitioned neutrosophic set, where each

$$M_i : U \rightarrow [0, 1] \quad (i = 1, \dots, k)$$

is a membership function and for all $x \in U$ the partition-sum bound

$$\sum_{i=1}^k M_i(x) \leq k$$

holds. We construct a plithogenic set

$$PS = (P, V, P_v, pdf, pCF)$$

that "realizes" N_k as follows:

1. Set $P := U$.
2. Choose $V := P_V := \{v_1, v_2, \dots, v_k\}$, one attribute value for each neutrosophic component.
3. Define the degree-of-appurtenance function

$$\text{pdf} : P \times P_V \longrightarrow [0, 1] \quad \text{by} \quad \text{pdf}(x, v_i) := M_i(x), \quad i = 1, \dots, k.$$

Then for each fixed $x \in U$, the k -tuple

$$(\text{pdf}(x, v_1), \dots, \text{pdf}(x, v_k)) = (M_1(x), \dots, M_k(x))$$

exactly reproduces the membership values of N_k .

4. Define the contradiction function to be trivial:

$$\text{pCF} : P_V \times P_V \longrightarrow [0, 1], \quad \text{pCF}(v_i, v_j) = 0 \quad \forall i, j.$$

This choice satisfies reflexivity and symmetry, and imposes no further restriction on the appurtenance degrees.

Since each $\text{pdf}(x, v_i) \in [0, 1]$ and pCF is well-defined and symmetric with zero diagonal, $(P, V, P_V, \text{pdf}, \text{pCF})$ is a valid plithogenic set. Moreover, the original k -partitioned neutrosophic set N_k is recovered by reading off $\text{pdf}(x, v_i)$ for $i = 1, \dots, k$.

Because k was arbitrary in $\{4, \dots, 10\}$, the same construction embeds every quadri-, penta-, hexa-, hepta-, octa-, nona-, and deca-partitioned neutrosophic set into a single unifying plithogenic framework. $\square$

From the above, the equivalence relations as shown in Tables 1 and 2 hold.

k	Partitioned Set	Attribute Values V
1	Fuzzy Set	$\{v_1\}$ corresponding to $\{\mu\}$
2	Intuitionistic Fuzzy Set	$\{v_1, v_2\}$ corresponding to $\{\mu, \nu\}$
3	Neutrosophic Set	$\{v_1, v_2, v_3\}$ corresponding to $\{T, I, F\}$
4	Quadripartitioned Neutrosophic Set	$\{v_1, \dots, v_4\}$ corresponding to $\{T, C, G, F\}$
5	Pentapartitioned Neutrosophic Set	$\{v_1, \dots, v_5\}$ corresponding to $\{T, C, G, U, F\}$
6	Hexapartitioned Neutrosophic Set	$\{v_1, \dots, v_6\}$ corresponding to $\{T, C, G, U, H, F\}$
7	Heptapartitioned Neutrosophic Set	$\{v_1, \dots, v_7\}$ corresponding to $\{T, M, C, U, I, K, F\}$
8	Octapartitioned Neutrosophic Set	$\{v_1, \dots, v_8\}$ corresponding to $\{T, M, C, U, I, K, H, F\}$
9	Nonapartitioned Neutrosophic Set	$\{v_1, \dots, v_9\}$ corresponding to $\{T, ST, WT, C, U, I, SF, WF, F\}$
10	Decapartitioned Neutrosophic Set	$\{v_1, \dots, v_{10}\}$ corresponding to $\{T, SRT, WRT, C, U, I, H, SRF, WRF, F\}$

Table 1: Embedding of k -partitioned fuzzy, intuitionistic fuzzy, and neutrosophic sets into a plithogenic set via attribute values V .

5 Conclusion and Future Works

In this paper, we formally introduced the notions of hexa-, octa-, nona-, and deca-partitioned neutrosophic sets, and demonstrated that each of these structures can be naturally embedded within the broader plithogenic set framework.

Looking ahead, several promising research directions emerge. Various generalizations and extensions of neutrosophic sets have been actively explored in the literature, including their applications to graph theory

s	t	Plithogenic Set Type
1	1	Plithogenic Fuzzy Set
2	1	Plithogenic Intuitionistic Fuzzy Set
3	1	Plithogenic Neutrosophic Set
4	1	Plithogenic Quadripartitioned Neutrosophic Set (cf.[86, 51, 87])
5	1	Plithogenic Pentapartitioned Neutrosophic Set (cf.[37, 88, 89])
6	1	Plithogenic Hexapartitioned Neutrosophic Set (cf.[69])
7	1	Plithogenic Heptapartitioned Neutrosophic Set(cf.[90, 91])
8	1	Plithogenic Octapartitioned Neutrosophic Set
9	1	Plithogenic Nonapartitioned Neutrosophic Set
10	1	Plithogenic Decapartitioned Neutrosophic Set

Table 2: Classification of Plithogenic Sets by number of attribute–component values (s) and auxiliary parameter (t).

[92, 93], hypergraphs [94, 95, 96], superhypergraphs [97, 98, 99, 100, 101], as well as offset structures [102, 103, 49], oversets [104, 105], undersets [106], and topological frameworks [107]. In future studies, we aim to investigate further extensions of the partitioned neutrosophic models—particularly along graph-based structures and offset-based generalizations—to deepen their theoretical development and broaden their applicability in complex systems modeling.

Acknowledgments

We extend our sincere gratitude to everyone who provided insights, inspiration, and assistance throughout this research. We particularly thank our readers for their interest and acknowledge the authors of the cited works for laying the foundation that made our study possible. We also appreciate the support from individuals and institutions that provided the resources and infrastructure needed to produce and share this paper. Finally, we are grateful to all those who supported us in various ways during this project.

Data Availability

Not applicable

Competing interests

Author has declared that no competing interests exist.

References

- [1] Lotfi A Zadeh. Fuzzy sets. *Information and control*, 8(3):338–353, 1965.
- [2] Talal Al-Hawary. Complete fuzzy graphs. *International Journal of Mathematical Combinatorics*, 4:26, 2011.
- [3] Azriel Rosenfeld. Fuzzy graphs. In *Fuzzy sets and their applications to cognitive and decision processes*, pages 77–95. Elsevier, 1975.
- [4] Krassimir T Atanassov and Krassimir T Atanassov. *Intuitionistic fuzzy sets*. Springer, 1999.
- [5] Krassimir T Atanassov. *On intuitionistic fuzzy sets theory*, volume 283. Springer, 2012.

- [6] Krassimir T Atanassov and G Gargov. *Intuitionistic fuzzy logics*. Springer, 2017.
- [7] Humberto Bustince and P Burillo. Vague sets are intuitionistic fuzzy sets. *Fuzzy sets and systems*, 79(3):403–405, 1996.
- [8] Muhammad Gulistan, Naveed Yaqoob, Ahmed Elmoasry, and Jawdat Alebraheem. Complex bipolar fuzzy sets: An application in a transport’s company. *J. Intell. Fuzzy Syst.*, 40:3981–3997, 2021.
- [9] Vicenç Torra and Yasuo Narukawa. On hesitant fuzzy sets and decision. In *2009 IEEE international conference on fuzzy systems*, pages 1378–1382. IEEE, 2009.
- [10] Vicenç Torra. Hesitant fuzzy sets. *International journal of intelligent systems*, 25(6):529–539, 2010.
- [11] Muhammad Akram, Danish Saleem, and Talal Al-Hawary. Spherical fuzzy graphs with application to decision-making. *Mathematical and Computational Applications*, 25(1):8, 2020.
- [12] Fatma Kutlu Gündoğdu and Cengiz Kahraman. Spherical fuzzy sets and spherical fuzzy topsis method. *Journal of intelligent & fuzzy systems*, 36(1):337–352, 2019.
- [13] Bui Cong Cuong and Vladik Kreinovich. Picture fuzzy sets-a new concept for computational intelligence problems. In *2013 third world congress on information and communication technologies (WICT 2013)*, pages 1–6. IEEE, 2013.
- [14] Sankar Das, Ganesh Ghorai, and Madhumangal Pal. Picture fuzzy tolerance graphs with application. *Complex & Intelligent Systems*, 8(1):541–554, 2022.
- [15] Muhammad Riaz, Muhammad Tahir Hamid, Deeba Afzal, Dragan Pamucar, and Yuming Chu. Multi-criteria decision making in robotic agri-farming with q-rung orthopair m-polar fuzzy sets. *PLoS ONE*, 16, 2021.
- [16] Muhammad Riaz and Masooma Raza Hashmi. Soft rough pythagorean m-polar fuzzy sets and pythagorean m-polar fuzzy soft rough sets with application to decision-making. *Computational and Applied Mathematics*, 39, 2019.
- [17] Juanjuan Chen, Shenggang Li, Shengquan Ma, and Xueping Wang. m-polar fuzzy sets: an extension of bipolar fuzzy sets. *The scientific world journal*, 2014(1):416530, 2014.
- [18] Florentin Smarandache. A unifying field in logics: Neutrosophic logic. In *Philosophy*, pages 1–141. American Research Press, 1999.
- [19] Haibin Wang, Florentin Smarandache, Rajshekhar Sunderraman, and Yan-Qing Zhang. *interval neutrosophic sets and logic: theory and applications in computing: Theory and applications in computing*, volume 5. Infinite Study, 2005.
- [20] Haibin Wang, Florentin Smarandache, Yanqing Zhang, and Rajshekhar Sunderraman. *Single valued neutrosophic sets*. Infinite study, 2010.
- [21] Takaaki Fujita and Florentin Smarandache. *Exploring Concepts of HyperFuzzy, HyperNeutrosophic, and HyperPlithogenic Sets (I)*. Infinite Study, 2025.
- [22] Takaaki Fujita and Iqbal M Batiha. Note of hyperneutrosophic crisp set and superhyperneutrosophic crisp set. *Neutrosophic Sets and Systems*, 96:119–132, 2026.
- [23] Takaaki Fujita. *Advancing Uncertain Combinatorics through Graphization, Hyperization, and Uncertainization: Fuzzy, Neutrosophic, Soft, Rough, and Beyond*. Biblio Publishing, 2025.
- [24] Zdzisław Pawlak. Rough sets. *International journal of computer & information sciences*, 11:341–356, 1982.
- [25] Said Broumi, Florentin Smarandache, and Mamoni Dhar. Rough neutrosophic sets. *Infinite Study*, 32:493–502, 2014.
- [26] Takaaki Fujita and Florentin Smarandache. A concise introduction to hyperfuzzy, hyperneutrosophic, hyperplithogenic, hypersoft, and hyperrough sets with practical examples. *Neutrosophic Sets and Systems*, 80:609–631, 2025.
- [27] Dmitriy Molodtsov. Soft set theory-first results. *Computers & mathematics with applications*, 37(4-5):19–31, 1999.
- [28] Abdallah Al-Husban, Maha Mohammed Saeed, Giorgio Nardo, Takaaki Fujita, Arif Mehmood Khattak, Raed Hatamleh, Ahmad A Abubaker, Jamil J Hamja, and Cris L Armada. A comprehensive study of bipolar vague soft expert p-open sets in bipolar vague soft expert topological spaces with applications to cancer diagnosis. *European Journal of Pure and Applied Mathematics*, 18(2):5900–5900, 2025.
- [29] Florentin Smarandache. Extension of soft set to hypersoft set, and then to plithogenic hypersoft set. *Neutrosophic sets and systems*, 22(1):168–170, 2018.
- [30] Sagvan Y Musa, Ramadhan A Mohammed, and Baravan A Asaad. N-hypersoft sets: An innovative extension of hypersoft sets and their applications. *Symmetry*, 15(9):1795, 2023.
- [31] Florentin Smarandache. *Plithogeny, plithogenic set, logic, probability, and statistics*. Infinite Study, 2017.
- [32] Florentin Smarandache. *Plithogenic set, an extension of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets-revisited*. Infinite study, 2018.

- [33] P Sathya, Nivetha Martin, and Florentine Smarandache. Plithogenic forest hypersoft sets in plithogenic contradiction based multi-criteria decision making. *Neutrosophic Sets and Systems*, 73:668–693, 2024.
- [34] Satham S Hussain, Muhammad Aslam, Hossein Rahmonlou, and N Durga. Applying interval quadripartitioned single-valued neutrosophic sets to graphs and climatic analysis. In *Data-Driven Modelling with Fuzzy Sets*, pages 100–143. CRC Press, 2025.
- [35] Arif Mehmood Khattak, M Arslan, Abdallah Shihadeh, Wael Mahmoud Mohammad Salameh, Abdallah Al-Husban Al-Husban, R Seethalakshmi, G Nordo, Takaaki Fujita, and Maha Mohammed Saeed. A breakthrough approach to quadri-partitioned neutrosophic softtopological spaces. *European Journal of Pure and Applied Mathematics*, 18(2):5845–5845, 2025.
- [36] Manal Al-Labadi, Shuker Khalil, VR Radhika, K Mohana, et al. Pentapartitioned neutrosophic vague soft sets and its applications. *International Journal of Neutrosophic Science*, (2):64–4, 2025.
- [37] Suman Das, Rakhal Das, and Binod Chandra Tripathy. Topology on rough pentapartitioned neutrosophic set. *Iraqi Journal of Science*, 2022.
- [38] T Mythili, V Jeyanthi, and KR Senthilkumar. Enhanced decision-making academic libraries with topsis methods using blockchain technology by heptapartitioned neutrosophic number. In *Leveraging Blockchain for Future-Ready Libraries*, pages 123–140. IGI Global Scientific Publishing, 2025.
- [39] Heng He. A novel approach to assessing art education teaching quality in vocational colleges based on double-valued neutrosophic numbers and multi-attribute decision-making with tree soft sets. *Neutrosophic Sets and Systems*, 78:206–218, 2025.
- [40] Qiuyan Zhao and Wentao Li. Incorporating intelligence in multiple-attribute decision-making using algorithmic framework and double-valued neutrosophic sets: Varied applications to employment quality evaluation for university graduates. *Neutrosophic Sets and Systems*, 76:59–78, 2025.
- [41] Qaisar Khan, Peide Liu, and Tahir Mahmood. Some generalized dice measures for double-valued neutrosophic sets and their applications. *Mathematics*, 6(7):121, 2018.
- [42] Ilanthenral Kandasamy. Double-valued neutrosophic sets, their minimum spanning trees, and clustering algorithm. *Journal of Intelligent systems*, 27(2):163–182, 2018.
- [43] Zhiheng Zhang. Enhanced decision-making technique for innovation capability evaluation in the core industries of digital economy under double-valued neutrosophic sets. *Neutrosophic Sets and Systems*, 77:492–509, 2025.
- [44] Yourong Guo. Enhancing dvnw-wcsm technique for double-valued neutrosophic multiple-attribute decision-making in digital economy: A case study on enhancing the quality of development of henan’s cultural and tourism industry. *Neutrosophic Sets and Systems*, 75:391–407, 2025.
- [45] Takaaki Fujita. Triple-valued neutrosophic set, quadruple-valued neutrosophic set, quintuple-valued neutrosophic set, and double-valued indetermssoft set. *Neutrosophic Systems with Applications*, 25(5):3, 2025.
- [46] Jiarong Jia. Triple-valued neutrosophic off for public digital cultural service level evaluation in craft and art museums. *Neutrosophic Sets and Systems*, 88(1):44, 2025.
- [47] Yuan Sun. Assessing the performance of basic education informatization in the digital age using quintuple-valued neutrosophic set. *Neutrosophic Sets and Systems*, 88(1):18, 2025.
- [48] Hongxin Wang. Professional identity formation in traditional chinese medicine students: An educational perspective using triple-valued neutrosophic set. *Neutrosophic Sets and Systems*, 88:83–92, 2025.
- [49] Florentin Smarandache. *Neutrosophic Overset, Neutrosophic Underset, and Neutrosophic Offset. Similarly for Neutrosophic Over-/Under-/Off-Logic, Probability, and Statistics*. Infinite Study, 2016.
- [50] R Radha, A Stanis Arul Mary, and Florentin Smarandache. Quadripartitioned neutrosophic pythagorean soft set. *International Journal of Neutrosophic Science (IJNS) Volume 14*, 2021, page 11, 2021.
- [51] Satham Hussain, Jahir Hussain, Isnaini Rosyida, and Said Broumi. Quadripartitioned neutrosophic soft graphs. In *Handbook of Research on Advances and Applications of Fuzzy Sets and Logic*, pages 771–795. IGI Global, 2022.
- [52] Bimal Shil¹, Rakhal Das, Carlos Granados, Suman Das, and Binod Chandra Tripathy. Single-valued quadripartitioned neutrosophic d-ideal of d-algebra. *Neutrosophic Sets and Systems*, vol. 67/2024: *An International Journal in Information Science and Engineering*, page 105, 2024.
- [53] Maha Mohammed Saeed, Raed Hatamleh, Alaa M Abdelatif, Abdallah Al-Husban Al-Husban, Husham M Attaalfadeel, Takaaki Fujita, Khaled A Aldwoah, and Arif Mehmood Khattak. A breakthrough approach to quadri-partitioned neutrosophic soft topological spaces. *European Journal of Pure and Applied Mathematics*, 18(2):5845–5845, 2025.

- [54] Rakhal Das and Suman Das. Pentapartitioned neutrosophic subtraction algebra. *Neutrosophic Sets and Systems*, 68:89–98, 2024.
- [55] Rama Mallick and Surapati Pramanik. *Pentapartitioned neutrosophic set and its properties*, volume 36. Infinite Study, 2020.
- [56] T Mythili, V Jeyanthi, D Maheswari, and WF Al Omeri. Heptapartitioned neutrosophic soft matrices and its application in medical diagnosis. *Neutrosophic Sets and Systems*, 97:456–479, 2026.
- [57] Raed Hatamleh, Nasir Odat, Abdallah Al-Husban, Arif Mehmood Khattak, Alaa M Abd El-latif, Husham M Attaalfadeel, Walid Abdelfattah, and Ahmad M Abdel-Mageed. Mapping love: A heptapartitioned neutrosophic machine learning study of university students’ romantic sensations. *European Journal of Pure and Applied Mathematics*, 18(3):6484–6484, 2025.
- [58] Florentin Smarandache. Neutrosophy: neutrosophic probability, set, and logic: analytic synthesis & synthetic analysis. 1998.
- [59] P Yiarayong. Some weighted aggregation operators of quadripartitioned single-valued trapezoidal neutrosophic sets and their multi-criteria group decision-making method for developing green supplier selection criteria. *OPSEARCH*, pages 1–55, 2024.
- [60] Bimal Shil, Rakhal Das, Carlos Granados, Suman Das, and Binod Chandra Tripathy. Single-valued quadripartitioned neutrosophic d-ideal of d-algebra. *Neutrosophic Sets and Systems*, 67:105–114, 2024.
- [61] Tanmoy Chatterjee and Surapati Pramanik. Triangular fuzzy quadripartitioned neutrosophic set and its properties. *Neutrosophic Sets and Systems*, 75:15–28, 2025.
- [62] Suman Das, Prasanna Poojary, Vadiraja Bhatta GR, and Sunny Kumar Sharma. Single-valued pentapartitioned neutrosophic bi-topological spaces. *International Journal of Neutrosophic Science (IJNS)*, 23(4), 2024.
- [63] M Myvizhi, Mohamed Elkholy, Ahmed Abdelhafeez, Hussam Elbehiery, et al. Enhanced madm strategy with heptapartitioned neutrosophic distance metrics. *Neutrosophic Sets and Systems*, 78:74–96, 2025.
- [64] T Mythili, V Jeyanthi, et al. Enhanced decision-making academic libraries with topsis methods using blockchain technology by heptapartitioned neutrosophic number. In *Leveraging Blockchain for Future-Ready Libraries*, pages 123–140. IGI Global Scientific Publishing, 2025.
- [65] Hussam Elbehiery. Enhanced madm strategy with heptapartitioned neutrosophic distance metrics. *Neutrosophic Sets and Systems*, vol. 78/2025: *An International Journal in Information Science and Engineering*, page 74, 2025.
- [66] Herbert F Weisberg and Jerrold G Rusk. Dimensions of candidate evaluation. *American Political Science Review*, 64(4):1167–1185, 1970.
- [67] Carolyn L Funk. Bringing the candidate into models of candidate evaluation. *The journal of politics*, 61(3):700–720, 1999.
- [68] Arden Rothstein. Fostering the educational value of candidate evaluation. *The International Journal of Psychoanalysis*, 98(6):1641–1668, 2017.
- [69] Vasile Patrascu. Penta and hexa valued representation of neutrosophic information. *arXiv preprint arXiv:1603.03729*, 2016.
- [70] Mumtaz Ali, Nguyen Dang Thanh, Nguyen Van Minh, et al. A neutrosophic recommender system for medical diagnosis based on algebraic neutrosophic measures. *Applied Soft Computing*, 71:1054–1071, 2018.
- [71] S Senthil Kumar and H Hannah Inbarani. Optimistic multi-granulation rough set based classification for medical diagnosis. *Procedia Computer Science*, 47:374–382, 2015.
- [72] Klaus-Peter Adlassnig. Fuzzy set theory in medical diagnosis. *IEEE Transactions on Systems, Man, and Cybernetics*, 16(2):260–265, 1986.
- [73] Lakshya Kumar, Apurva Jindal, and Nagendra Rao Velaga. Financial risk assessment and modelling of ppp based indian highway infrastructure projects. *Transport Policy*, 62:2–11, 2017.
- [74] Muhammad Junaid, Ye Xue, Muzzammil Wasim Syed, Ji Zu Li, and Muhammad Ziaullah. A neutrosophic ahp and topsis framework for supply chain risk assessment in automotive industry of pakistan. *Sustainability*, 2019.
- [75] Qi Zhang, Chunjie Zhou, Yu-Chu Tian, Naixue N. Xiong, Yuanqing Qin, and Bowen Hu. A fuzzy probability bayesian network approach for dynamic cybersecurity risk assessment in industrial control systems. *IEEE Transactions on Industrial Informatics*, 14:2497–2506, 2018.
- [76] Saeed Samiee. Customer evaluation of products in a global market. *Journal of international business studies*, 25(3):579–604, 1994.
- [77] Gilbert A Churchill Jr and Carol Surprenant. An investigation into the determinants of customer satisfaction. *Journal of marketing research*, 19(4):491–504, 1982.
- [78] Arch G Woodside, Lisa L Frey, and Robert Timothy Daly. Linking service quality, customer satisfaction, and behavior. *Marketing Health Services*, 9(4):5, 1989.

- [79] Sedat Belbag, Aybegum Gungordu, Tuba Yumusak, and Kadri Gökhan Yılmaz. The evaluation of smartphone brand choice: an application with the fuzzy electre i method. *International Journal of Business and Management Invention*, 5(3):55–63, 2016.
- [80] Shankha Shubhra Goswami and Dhiren Kumar Behera. Evaluation of the best smartphone model in the market by integrating fuzzy-ahp and promethee decision-making approach. *Decision*, 48:71–96, 2021.
- [81] Takaaki Fujita and Florentin Smarandache. *Plithogenic Superhypersoft Set and Plithogenic Forest SuperHypersoft Set*. Infinite Study, 2025.
- [82] Florentin Smarandache. Plithogeny, plithogenic set, logic, probability, and statistics. *arXiv preprint arXiv:1808.03948*, 2018.
- [83] Paul Lyons. A robust approach to employee skill and knowledge development. *Industrial and Commercial Training*, 37(1):3–9, 2005.
- [84] Nicole Gravina, Jessica Nastasi, and John Austin. Assessment of employee performance. *Journal of Organizational Behavior Management*, 41(2):124–149, 2021.
- [85] Mohamad Sattar Rasul, Rose Amnah Abd Rauf, Azlin Norhaini Mansor, and AP Puvanasvaran. Employability skills assessment tool development. *International Education Studies*, 5(5):43–56, 2012.
- [86] M Ramya, Sandesh Murali, and R.Radha. Bipolar quadripartitioned neutrosophic soft set. 2022.
- [87] Xiaolong Shi, Saeed Kosari, Hossein Rashmanlou, Said Broumi, and S Satham Hussain. Properties of interval-valued quadripartitioned neutrosophic graphs with real-life application. *Journal of Intelligent & Fuzzy Systems*, 44(5):7683–7697, 2023.
- [88] Rama Mallick and Surapati Pramanik. Pentapartitioned neutrosophic set and its properties. *Neutrosophic Sets and Systems*, 35:49, 2020.
- [89] Pranab Biswas, Surapati Pramanik, and Bibhas Chandra Giri. Single valued bipolar pentapartitioned neutrosophic set and its application in madm strategy. 2022.
- [90] M Myvizhi, Ahmed M Ali, Ahmed Abdelhafeez, and Haitham Rizk Fadlallah. *MADM Strategy Application of Bipolar Single Valued Heptapartitioned Neutrosophic Set*. Infinite Study, 2023.
- [91] S Broumi and Tomasz Witczak. Heptapartitioned neutrosophic soft set. *International Journal of Neutrosophic Science*, 18(4):270–290, 2022.
- [92] Said Broumi, Mohamed Talea, Assia Bakali, and Florentin Smarandache. Single valued neutrosophic graphs. *Journal of New theory*, (10):86–101, 2016.
- [93] Reinhard Diestel. *Graph theory*. Springer (print edition); Reinhard Diestel (eBooks), 2024.
- [94] Claude Berge. *Hypergraphs: combinatorics of finite sets*, volume 45. Elsevier, 1984.
- [95] Alain Bretto. Hypergraph theory. *An introduction. Mathematical Engineering. Cham: Springer*, 1, 2013.
- [96] Yifan Feng, Haoxuan You, Zizhao Zhang, Rongrong Ji, and Yue Gao. Hypergraph neural networks. In *Proceedings of the AAAI conference on artificial intelligence*, volume 33, pages 3558–3565, 2019.
- [97] Florentin Smarandache. *Extension of HyperGraph to n-SuperHyperGraph and to Plithogenic n-SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical-/Neuro-/Anti-) HyperAlgebra*. Infinite Study, 2020.
- [98] Mohammed Alqahtani. Intuitionistic fuzzy quasi-supergraph integration for social network decision making. *International Journal of Analysis and Applications*, 23:137–137, 2025.
- [99] N. B. Nalawade, M. S. Bapat, S. G. Jakkewad, G. A. Dhanorkar, and D. J. Bhosale. Structural properties of zero-divisor hypergraph and superhypergraph over $\mathbb{Z}_n$: Girth and helly property. *Panamerican Mathematical Journal*, 35(4S):485, 2025.
- [100] Takaaki Fujita and Arif Mehmood. Fuzzy tolerance hypergraphs and fuzzy tolerancesuperhypergraphs. *Neutrosophic Computing and Machine Learning. ISSN 2574-1101*, 41:46–70, 2025.
- [101] Eduardo Martín Campoverde Valencia, Jessica Paola Chuisaca Vásquez, and Francisco Ángel Becerra Lois. Multineutrosophic analysis of the relationship between survival and business growth in the manufacturing sector of azuay province, 2020–2023, using plithogenic n-superhypergraphs. *Neutrosophic Sets and Systems*, 84:341–355, 2025.
- [102] Roger Martinez Isaac, Rudy García Cobas, Carlos Roberto Humanante Cabrera, María Andrea Soto Granda, and Linda Florangely Soto Torres. Empowering multigrade interdisciplinary assessment through plithogenic offsets and artificial intelligence for dynamic and personalized education. *Neutrosophic Sets and Systems*, 92:398–412, 2025.
- [103] Génesis Del Carmen Alcívar Junco. Identification of gaps in ecuadorian inclusive university education through plithogenic offsets. *Neutrosophic Sets and Systems*, 92:510–522, 2025.
- [104] Florentin Smarandache. Interval-valued neutrosophic oversets, neutrosophic undersets, and neutrosophic offsets. *Collected Papers. Volume IX: On Neutrosophic Theory and Its Applications in Algebra*, page 117, 2022.

- [105] Nivetha Martin, Priya Priya.R, and Florentin Smarandache. Decision making on teachers' adaptation to cybergogy in saturated interval- valued refined neutrosophic overset /underset /offset environment. *International Journal of Neutrosophic Science*, 2020.
- [106] Florentin Smarandache. Operators on single-valued neutrosophic oversets, neutrosophic undersets, and neutrosophic offsets. *Collected Papers*, 9:112, 2022.
- [107] Fatimah M. Mohammed, Anas A. Hijab, and Shaymaa F. Matar. Fuzzy neutrosophic weakly-generalized closed sets in fuzzy neutrosophic topological spaces. *Journal of University of Anbar for Pure Science*, 2022.